

## Finite Mathematical Structures A

Exam 2: Wednesday, November 11, 2009

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 5 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed.

There may be problems that are solvable by inspection, but if you get the wrong answer and have shown no work, then I will assign NO partial credit.

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - turn them off AND put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

**Last (Family) Name (PRINT CLEARLY):** \_\_\_\_\_

**First Name (PRINT CLEARLY):** \_\_\_\_\_

**ID Number:** \_\_\_\_\_

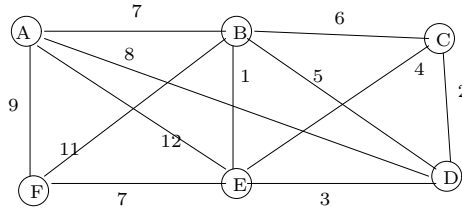
Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

**Signature:**

\_\_\_\_\_

1. (5 points) Let  $T$  be a 6-ary tree with 100 internal nodes. How many leaves does the tree have? (A correct guess with no work shown will receive very partial credit.)

2. (15 points) Consider the following graph.



(a). Highlight the edges of a minimum spanning tree of the graph.

(b). The cost of every edge in the graph is multiplied by 10. Circle the correct answers:

Does this change the edges in the minimum spanning tree?    YES    NO

Does this change the cost of the minimum spanning tree?    YES    NO

(c). Ignoring the costs on the edges, show the BFS tree of the graph starting at node A.

3. (8 points) We wish to model the word problem as a graph problem: The Happy Gardner, a lawnmowing service, has been contracted to mow the lawns at 15 houses today. The gardener would like to plan his schedule for the day, namely, in what order the will the different lawns be mowed. We may assume the gardener starts his day at his house, and would like to return home as soon as possible. We also assume travel times between houses are known.

(a). What do the nodes of the graph to be constructed represent?

(b). What do the edges of the graph to be constructed represent?

(c). State which graph problem it is: Shortest path, Minimum spanning tree, Traveling Salesman Problem, Breadth First Search or Depth First Search.

4. (12 points) True or False? If true, give a short proof. If false, give a counterexample: We are given a connected graph  $G$  with costs on edges. Assume all costs are positive and that there are no ties.

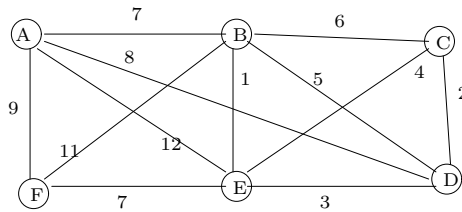
(a). If edge  $e = (A, B)$  is the cheapest edge touching node A, then it must be part of a Minimum Spanning Tree.

(b). If edge  $e = (A, B)$  is the cheapest edge touching node A, then it must be part of a shortest path tree rooted at node A.

(c). If edge  $e = (A, B)$  is the cheapest edge touching node B, then it must be part of a shortest path tree rooted at node A.

(d). If edge  $e = (B, C)$  is part of a Minimum Spanning Tree then it must also be part of a shortest path tree rooted at node A.

5. (10 points) (a). Highlight the edges of the shortest path tree rooted at node  $D$  for the graph below.



(b). The cost of every edge in the graph is multiplied by 10. Circle the correct answers:

Does this change the edges in the shortest path tree?      YES      NO

Does this change the cost of the shortest path from node  $D$  to node  $A$ ?      YES      NO

6. (10 points) There are 100 (distinct) students at a school and 3 dormitories, A,B,C, with capacities 25, 35, and 40 respectively.

(a). How many ways are there to fill the dormitories?

(b). Suppose that of the 100 students 50 are men and 50 are women and that A is an all-men's dorm, B is an all-women's dorm and C is co-ed. How many ways are there to fill the dormitories?

(7).(5 points) Determine the number of ways to distribute 10 identical orange drinks, 1 lemon drink, and 1 lime drink to 4 thirsty (distinct!) children so that each child gets at least one drink, and the lemon and lime drinks go to different children.

(8). (15 points) (a). How many distinct arrangements of the letters in "PREPOSTEROUS" are there?

(b). How many distinct arrangements of the letters in "PREPOSTEROUS" are there, in which the first "E" precedes the first "O"?

(c). How many distinct arrangements of the letters in "PREPOSTEROUS" are there, in which the 5 vowels are consecutive?

(9). (20 points) Of a company's personnel, 7 work in design, 14 in manufacturing, 4 in testing, 5 in sales, 2 in accounting, and 3 in marketing. (Each of the 35 workers works in a single department.) A committee of 6 people is to be formed to meet with upper management. Answer each part separately.

(a). In how many ways can the committee be formed if there is to be exactly one member of each department?

(b). In how many ways can the committee be formed if there must be exactly two members from the manufacturing department?

(c). In how many ways can the committee be formed if there must be at least two members from the manufacturing department?

(d). Lucy works in the design department, her husband Rickey works in marketing. In how many ways can the committee be formed if they cannot both be on the committee together? (It is ok for either Lucy or Rickey or neither of them to be on the committee.)