

## Finite Mathematical Structures A

Exam 2: Thursday, April 8, 2010

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 6 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed.

There may be problems that are solvable by inspection, but if you get the wrong answer and have shown no work, then I will assign NO partial credit.

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - turn them off AND put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

**Last (Family) Name (PRINT CLEARLY):** \_\_\_\_\_

**First Name (PRINT CLEARLY):** \_\_\_\_\_

**ID Number:** \_\_\_\_\_

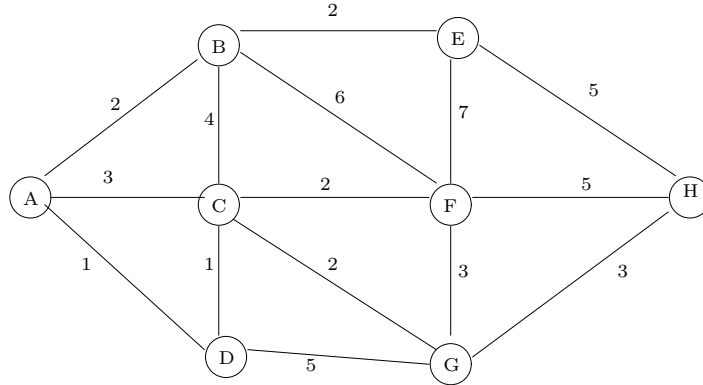
Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

**Signature:**

\_\_\_\_\_

1. (5 points) Let  $T$  be a 4-ary tree with 200 internal nodes. How many leaves does the tree have?  
 (A correct guess with no work shown will receive very partial credit.)

2. (9 points) Consider the following graph.



(a). Highlight the edges of a minimum spanning tree of the graph.

(b). Edge (A,B) is currently part of the MST, however its cost is uncertain. What are all the possible costs of the edge for which it will be part of the MST? Explain briefly. (Your answer should be of the sort cost is greater than 3 and less than or equal to 17.)

3. (20 points) True or False? Make sure to explain! (The best explanation if true, is a short proof, and if false, a counterexample.) We are given a connected graph  $G$  with costs on edges. Assume all costs are positive and that there are no ties.  $A$  and  $B$  are two of the nodes in the graph.

(a). A BFS on  $G$  rooted at node  $A$  has the same number of edges as a BFS on  $G$  rooted at node  $B$ .

(b). A BFS on  $G$  rooted at node  $A$  has the same number of leaves as a BFS on  $G$  rooted at node  $B$ .

(c). I calculate an MST using Prim's algorithm, starting with node  $A$ , and Tiffany calculates an MST using Prim's algorithm, starting with node  $B$ . We must obtain the same tree.

(d). Tiffany and I each calculate a quick TSP, using either of the algorithms discussed in class. We must obtain the same tour.

(e). I calculate a the shortest path tree rooted at  $A$ , and Tiffany calculates the shortest path tree rooted at  $B$ . We must obtain the same shortest path tree.

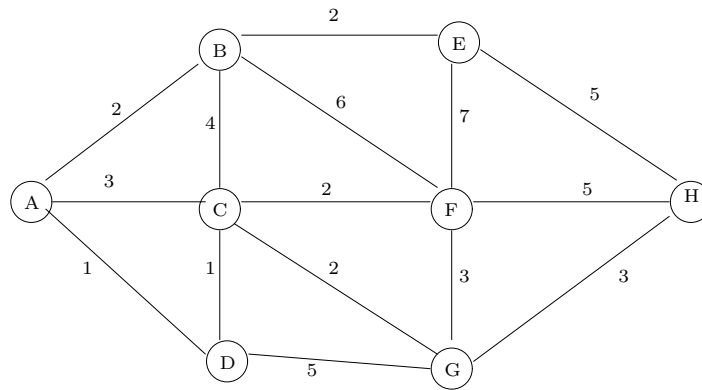
4. (6 points) We wish to model the word problem as a graph problem: The campus mail service must deliver interdepartmental mail to various buildings on campus. For simplicity, assume there is a single mailman who starts and ends his day at the central services building, and that he can carry all the mail to be delivered today in his cart at once. We also assume that we know how long it takes the mailman to walk between any two buildings.

(a). What do the nodes of the graph to be constructed represent?

(b). What do the edges of the graph to be constructed represent?

(c). State which graph problem it is: Shortest path, Minimum spanning tree, Traveling Salesman Problem, Breadth First Search or Depth First Search.

5. (10 points) (a). Highlight the edges of the shortest path tree rooted at node  $C$  for the graph below.



(b). Edge  $(C,F)$  is currently part of the shortest path tree, however its length is uncertain. What are all the possible lengths of the edge for which it will be part of the shortest path tree rooted at  $C$ ? Explain briefly. (Your answer should be of the sort cost is greater than 3 and less than or equal to 17.)

6. (8 points) We wish to arrange the digits 1,2,3,4,5,6,7,8,9.

(a). How many arrangements are possible?

(b). How many arrangements have no adjacent odd digits? For example, the arrangement 385164927 is not allowed because 5 and 1 are adjacent.

(7). (8 points) You throw four identical dice (each dice is 6 sided and shows the digits 1 through 6). You write down the values showing in nondecreasing order from left to right. For example 2245 means you rolled two 2s, one 4 and one 5.

(a). How many outcomes are possible?

(b). How many outcomes are possible in which all the values are different?

(8). (14 points) Two baseball teams A and B, play each other in a best of seven series (so that the first team to win 4 games wins the series). For example, the outcome ABAAA means that team B won the second game, team A won games 1,3,4,5 and therefore the series. Note that the series ends as soon as a team wins 4 games, and each game ends with one of the two teams winning (no ties).

(a). How many different outcomes are there if the series ends in exactly 6 games?

(b). How many different outcomes are there?

(9). (20 points) A 5 card hand is chosen at random from a standard deck of 52 cards. A standard deck has 13 cards from each of 4 suits (clubs, diamonds, hearts and spades).

(a). What is the probability that the hand contains a royal flush (Ace, King, Queen, Jack, 10 all in one suit)?

(b). What is the probability that the hand contains exactly 3 jacks and exactly 2 clubs?

(c). What is the probability that the hand contains a pair (that is exactly two cards of the same denomination and three other cards that do not contain a pair, for example 2 jacks a four a queen and a nine)?