

Finite Mathematical Structures A

Exam 3: Tuesday, May 11, 2010, 5:15-6:46pm

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 6 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. You may leave numerical answers in raw form, without evaluating them (e.g., $(1 + 2 + \cdots + 22)^3 - \sum_{i=1}^{12} \binom{17-i}{5} \binom{5}{4}$ can be left as is).

This examination is **CLOSED BOOK** and **CLOSED NOTES**. **Calculators are NOT allowed**. You may use a 4 by 6 “cheat sheet”, which should be turned in with your exam.

The following items should **NOT** be on your desk - turn them off **AND** put them **INSIDE** your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and **GOOD LUCK!!!**

Last (Family) Name (PRINT CLEARLY): _____

First Name (PRINT CLEARLY): _____

ID Number: _____

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature:

1. (20 points) Build a generating function for a_r in the following procedures. Remember to state which coefficient solves the initial problem. You do **not** need to calculate the coefficient.

(a). An exam has 20 questions worth 5 points each. Each problem receives 0,1,2,3,4,5 points. How many ways are there for a student to score r points? (Note that we do not care how many points any specific question receives, just the number of questions that receive 0 points, 1 point etc.)

(b). How many ways are there to distribute r identical forks to 10 people so that each person receives either one or two forks?

(c). The number of ways a team can score r points in a basketball game? (In basketball, any single shot is worth either one, two, or three points. We are only interested in the number of each type of shot made, not the order in which they were made.)

(d). In how many ways can we make change for r cents using 5 pennies, 3 nickels, and one dime?

2. (5 points) Solve the following recurrence relation: $a_n = 3a_{\frac{n}{3}} + 4$, $a_1 = 1$
(You may assume that $n = 3^m$, for some $m = 0, 1, 2, \dots$)

3. (10 points) Consider the recurrence $a_n = a_{n-1} + n$, $a_1 = 1$.

(a). Calculate a_2 and a_3 .

(b). Solve the recurrence relation. Make sure to verify your answer using induction.

4. (15 points) (a). What is the coefficient of x^{50} in the expansion of $(x^9 + x^{10} + x^{11} + \dots)^3$?

(b). What is the coefficient of x^{14} in the expansion of $(1 + x + x^4)/(1 - x)^5$?

5. (10 points) A produce stand sells only broccoli, carrots and okra. One day the stand served 207 customers. 114 people purchased broccoli, 152 purchased carrots, 25 purchased okra, 64 purchased broccoli and carrots, 12 purchased carrots and okra and 9 purchased all three vegetables. How many people purchased broccoli and okra?

6. (20 points) A fast food outlet gives away 4 different toys in children's meal packs, one toy per pack. If we buy 10 children's meal packs, what is the probability of getting all 4 toys?

7. (20 points) (a). Write a recurrence relation for a_n the number of n digit binary sequences with at least one instance of consecutive 0s. (You do not have to solve the recurrence.)

(b). Write a complete set of initial conditions for your recurrence in part (a).

(c). Calculate a_3 and a_4 using your recursion from (a) and initial conditions from (b).

(d). Write a recurrence relation for a_n the number of n digit ternary (0,1,2) sequences with at least one instance of consecutive 0s. (You do not have to solve the recurrence.)

(e). Write a complete set of initial conditions for your recurrence in part (d).