AMS 341 (Spring, 2008) Estie Arkin

Operations Research I: Deterministic Models

Exam 1: Thursday, March 13, 2008

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 6 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. (Use back of pages also for scratch paper if you need it.)

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

Last (Family) Name (PRINT CLEARLY): _____

First Name (PRINT CLEARLY): _____

ID Number: _____

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature:

_____

1. (18 points) Consider the following constraints for an LP:

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 5 \\ 2x_1 + x_2 + 6x_3 + x_4 &= 10 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

(a). Find a BFS (basic feasible solution) for this feasible region. Make sure to state which variables are basic and which are non basic!

(b). Find a Basic Solution that is not feasible for this feasible region.

(c). Find a degenerate BFS (basic feasible solution) for this feasible region.

2. (26 points) A farmer has a 200 acre farm on which he grows wheat, alfalfa, and beef. Wheat sells for $30 per bushel, alfalfa sells for $ 200 per bushel, and beef sells for $ 300 per ton. Upto 1,000 bushels of wheat and upto 1,000 bushels of alfalfa can be sold. Planting wheat on 1 acre yields 50 bushels, and requires 30 hours of labour. Planting alfalfa on 1 acre yields 100 bushels, and requires 20 hours of labour. Each acre of the farm devoted to raising beef yields 10 tons of beef and requires 50 hours of labour and 5 bushels of alfalfa. Upto 2,000 hours of labour are available. The farmer wants to maximize his profit (revenue minus costs) and asks that you formulate an LP to do so. (Your formulation does NOT have to be put into standard form. Do NOT solve, just formulate!)
(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

3. (18 points) Consider the LP: (It may be helpful to sketch it.) c_1 and c_2 are some constants.

$$\begin{array}{lll} \min & z = c_1x_1 + c_2x_2 & \\ \text{s.t.} & 3x_1 + 3x_2 & \geq 6 \\ & x_1 & \leq 5 \\ & x_1, x_2 & \geq 0 \end{array}$$

(a). Give an example for c_1, and c_2 such that the LP has a unique optimal solution.

(b). Give an example for c_1, and c_2 such that the LP is unbounded (optimal $z = -\infty$).

(c). Add a constraint to the problem so the LP has no feasible solutions.

4. (18 points) A company manufactures and sells dog food of two types. Each bag of type 1 dog food contains 2 pds of lamb and 4 pds of corn, and sells for \$5. Each bag of type 2 dog food contains 1 pd of corn and 1 pd of lamb, and sells for \$2. A total of 30 pds of lamb and 50 pds of corn are available. The company manager requires that at least 11 bags of dog food 1 are produced. Let $D1, D2$ be the number of bags of dog food type 1,2 produced. Use the Lindo output below to answer each of the following, or explain why you cannot give an answer without rerunning Lindo:

$$\begin{array}{rcl} \max & 5D1 + 2D2 & \\ \text{s.t. } 2) & 2D1 + D2 & \leq 30 \\ 3) & 4D1 + D2 & \leq 50 \\ 4) & D1 & \geq 11 \end{array}$$

objective function value 67.0000000

variable	value	reduced cost
$D1$	11.000000	.000000
$D2$	6.000000	.000000

row	slack or surplus	dual prices
2)	2.000000	0.000000
3)	.000000	2.000000
4)	.000000	−3.000000

Range in which basis remains unchanged :

OBJ coefficient ranges

variable	current coef	allowable increase	allowable decrease
$D1$	5.000000	3.000000	infinity
$D2$	2.000000	infinity	.750000

righthand side ranges

row	current RHS	allowable increase	allowable decrease
2	30.000000	infinity	2.000000
3	50.000000	2.000000	6.000000
4	11.000000	1.500000	1.000000

(a). If 40 pounds of corn were available (instead of 50), what would be the new optimal solution to the problem (the z)?

(b). What is the most that the company should be willing to pay to for another pound of corn?

(c). The company just learned that each bag of dog food 1 can sell for \$6 (instead of \$5). What would be the new optimal solution to the problem (the z)?

5. (20 points) We wish to solve the LP below using the big M method.

$$\begin{array}{lll} \min & z = 2x_1 + x_2 & \\ \text{s.t.} & 3x_1 + 3x_2 & \geq 6 \\ & x_1 & \leq 5 \\ & x_1, x_2 & \geq 0 \end{array}$$

(a). Rewrite the LP in standard form.

(b). Add artificial variable(s) and state the first tableau that will be used by the big M method (after "clean-up", eliminating the basic variables from the objective function)

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