1. (15 points) To supplement my income I plan to bake some cakes to sell at the Farmer’s Market. Each chocolate cake sells for $10 and each vanilla cake for $7.50. A chocolate cake requires 4 eggs and 40 minutes of baking time. Each vanilla cake requires 1 egg and 50 minutes of baking time. Assume at most 1 cake fits in my oven at any time. I currently have 30 eggs and 8 hours of baking time.

Assuming that I can sell all cakes that I bring to the Farmer’s Market, formulate an LP to maximize my profit:
(a). Define the variables you are using in the formulation. \( c \) = number of chocolate cakes baked, \( v \) = number of vanilla cakes baked.
(b). The objective function is: \( \max z = 10c + 7.5v \)
(c). The constraints are:

\[
4c + 1v \leq 30
\]
\[
40/60c + 50/60v \leq 8
\]
\( c, v \geq 0 \)

2. (12 points) The following is a tableau for an LP which is a MAX problem:

\[
\begin{array}{cccccc|c|c}
 & z & s_1 & x_1 & x_2 & e_3 & x_3 & \text{RHS} & \text{ratio} \\
1 & 1 & 0 & 2 & -1 & 0 & 0 & 12 & \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 4 & \\
0 & 0 & 2 & 0 & 1 & 0 & 3 & \\
0 & 1 & 1 & -3 & 0 & 0 & 6 & \\
\end{array}
\]

(a). What are the basic variables, and what are they equal to? \( s_1 = 6, e_3 = 3, x_3 = 4 \) \((z = 12)\)
(b). What are the non-basic variables, and what are they equal to? \( x_1 = x_2 = 0 \)
(c). This is not an optimal BFS. Which variable should be selected to enter the basis? \( x_2 \) enters, \( x_3 \) leaves.
(d). Do one pivot. What is the next tableau?

\[
\begin{array}{cccccc|c|c}
 & z & s_1 & x_1 & x_2 & e_3 & x_3 & \text{RHS} & \text{ratio} \\
1 & 1 & 0 & 2 & 0 & 0 & 1 & 16 & \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 4 & \\
0 & 0 & 2 & 0 & 1 & 0 & 3 & \\
0 & 1 & 1 & 0 & 0 & 3 & 18 & \\
\end{array}
\]

3. (20 points)(a) Consider the feasible region given by the following constraints: Sketch the feasible region.

\[
x_1 \leq 5 \quad (1)
\]
\[
x_2 \leq 4 \quad (2)
\]
\[
x_1 + x_2 \leq 8 \quad (3)
\]
In standard form:

\begin{align*}
x_1 + x_2 & \geq 2 \\
x_1 & \geq 0 \\
x_2 & \geq 0
\end{align*}

In parts (b), (c), (d) circle the correct answers:

(b). Is the point \( x_1 = 4, x_2 = 0 \) a feasible point? YES
Is it a basic solution? NO

c. Is the point \( x_1 = 5, x_2 = 0 \) a feasible point? YES
Is it a basic solution? YES

d. Is the point \( x_1 = 0, x_2 = 0 \) a feasible point? NO
Is it a basic solution? YES

e. How many feasible solutions does the LP have? \( \infty \)
(f). How many basic feasible solutions does the LP have? 6

4. (10 points) Consider the following LP:

\[
\begin{align*}
\min \quad & z = 2x - y + 2w \\
\text{s.t.} \quad & x + y \quad \geq 10 \\
& x - y + 3w = -2 \\
& x \quad \leq 0 \\
& y \quad \geq 0 \\
& w \quad \text{unrestricted}
\end{align*}
\]
Rewrite the LP in standard form.

\[
\begin{align*}
\text{min } & \quad z = -2x' - y + 2w_1 - 2w_2 \\
\text{s.t. } & \quad -x' + y - e_1 = 10 \\
& \quad -x' + y - 3w_1 + 3w_2 = 2 \\
& \quad x', y, w_1, w_2, e_1 \geq 0
\end{align*}
\]

5. (5 points) A maximization LP is being solved by the Simplex method. Here is the current tableau:

<table>
<thead>
<tr>
<th></th>
<th>z</th>
<th>1</th>
<th>0</th>
<th>3</th>
<th>0</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which one of the following statements is true: (Circle one)

(i). This is an optimal tableau, and the LP has a unique optimal solution.

6. (20 points) Steelco manufacturer steel by combining Alloy 1 and Alloy 2. The steel must meet the following requirements: 3.2-3.5% carbon (i.e., at least 3.2 but no more than 3.5 percent); 1.8-2.5% silicon; 0.9-1.2% nickel; tensile strength of at least 45,000 pounds per square inch (psi). Assume that the tensile strength of a mixture of the two alloys is determined by averaging that of the two alloys mixed. For example a one ton mixture that is 40% Alloy 1 and 60% Alloy 2 has tensile strength of 0.4(42,000) + 0.6(50,000). The cost and properties of the alloys are given below:

<table>
<thead>
<tr>
<th></th>
<th>Cost per ton</th>
<th>percent silicon</th>
<th>percent nickel</th>
<th>percent carbon</th>
<th>tensile strength (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alloy 1</td>
<td>$190</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>42,000</td>
</tr>
<tr>
<td>Alloy 2</td>
<td>$200</td>
<td>2.5</td>
<td>1.5</td>
<td>4</td>
<td>50,000</td>
</tr>
</tbody>
</table>

Formulate an LP to minimize the cost of producing one ton of steel.

(a). Define the variables you are using in the formulation. \( A_1, A_2 \) amount of alloy 1,2 bought.

(b). The objective function is: \( \text{min } z = 190A_1 + 200A_2 \)

(c). The constraints are:

\[
\begin{align*}
0.02A_1 + 0.025A_2 & \leq 0.025 \\
0.02A_1 + 0.025A_2 & \geq 0.018 \\
0.01A_1 + 0.015A_2 & \leq 0.012 \\
0.01A_1 + 0.015A_2 & \geq 0.009 \\
0.03A_1 + 0.04A_2 & \leq 0.035 \\
0.03A_1 + 0.04A_2 & \geq 0.032 \\
42A_1 + 50A_2 & \geq 45 \\
A_1 + A_2 & = 1 \\
A_1, A_2 & \geq 0
\end{align*}
\]
7. (18 points, 3 points for each part) Answer TRUE or FALSE:

**False** All optimal solutions to an LP must be basic (a BFS).

**True** Any BFS can be an optimal solution to an LP depending on the choice of objective function.

**True** Two different BFS (to the same LP) must have the same number of non basic variables.

**False** At the end of the Simplex method, all slack variables must equal zero.

**False** There is only one correct way to formulate an LP.

**True** An LP in cannonical form is also in standard form.