AMS 341 (Spring, 2008) Exam 1 - Solution notes Estie Arkin

Mean 82.28, median 85, high 100 (3 of them!), low 32.

1. (18 points) Consider the following constraints for an LP:

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 5 \\ 2x_1 + x_2 + 6x_3 + x_4 &= 10 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Recall definition of basic solution (see page 131 text): Set $n - m$ non basic variables equal to zero, and solve the equations for remaining m basic variables. There are $m = 2$ equality constraints and $n = 4$ variables, so we should have 2 basic and 2 non basic variables. We get the following basic solutions:

Basic Varibles	basicsolution	$Feasible?Degenerate?$	
x_3, x_4	$x_1 = 0, x_2 = 0, x_3 = 1, x_4 = 4$	feasible	nondegenerate
x_2, x_4	$x_1 = 0, x_2 = -5, x_3 = 0, x_4 = 15$	notfeasible	nondegenerate
x_2, x_3	$x_1 = 0, x_2 = 20/11, x_3 = 15/11, x_4 = 0$	feasible	nondegenerate
x_1, x_4	$x_1 = 5, x_2 = 0, x_3 = 0, x_4 = 0$	feasible	degenerate
x_1, x_2	$x_1 = 5, x_2 = 0, x_3 = 0, x_4 = 0$	feasible	degenerate
x_1, x_3	$x_1 = 5, x_2 = 0, x_3 = 0, x_4 = 0$	feasible	degenerate

(a). Find a BFS (basic feasible solution) for this feasible region. Make sure to state which variables are basic and which are non basic! One such example is: $x_3 = 1$, $x_4 = 4$ basic and $x_1 = x_2 = 0$ non basic.

(b). Find a Basic Solution that is not feasible for this feasible region. $x_2 = -5$, $x_4 = 15$ basic, $x_1 = x_3 = 0$ non basic..

Common mistake, having variable values that do not satisfy the equality constraints. This would not be a basic solution.

(c). Find a degenerate BFS (basic feasible solution) for this feasible region. $x_1 = 5$, $x_2 = 0$ basic, $x_3 = x_4 = 0$ non basic..

2. (26 points) A farmer has a 200 acre farm on which he grows wheat, alfalfa, and beef. Wheat sells for \$30 per bushel, alfalfa sells for \$ 200 per bushel, and beef sells for \$ 300 per ton. Upto 1,000 bushels of wheat and upto 1,000 bushels of alfalfa can be sold. Planting wheat on 1 acre yields 50 bushels, and requires 30 hours of labour. Planting alfalfa on 1 acre yields 100 bushels, and requires 20 hours of labour. Each acre of the farm devoted to raising beef yields 10 tons of beef and requires 50 hours of labour and 5 bushels of alfalfa. Upto 2,000 hours of labour are available. The farmer wants to maximize his profit (revenue minus costs) and asks that you formulate an LP to do so. (Your formulation does NOT have to be put into standard form. Do NOT solve, just formulate!)

(a). Define the variables you are using in the formulation. W = acres devoted to wheat, A = acres devoted to alfalfa, B = acres devoted to beef, AS = bushels of alfalfa sold, AB = bushels of alfalfa eaten by the beef.

(b). The objective function is: Max $z = 1500W + 200AS + 3000B$

Common mistake, subtracting costs. In this particular problem the costs are zero.

(c). The constraints are:

$$50W \leq 1000$$

$$AS \leq 1000$$
$$AS + AB = 100A$$
$$AB = 5B$$
$$W + B + A \leq 200$$
$$30W + 20A + 50B \leq 2000$$
$$W, B, A, AS, AB \geq 0$$

Common mistakes: defining a variable for alfalfa but not specifying whether it is grown or sold (and using it both ways!), defining variables and not connecting them, such as hours to grow wheat, and a variable for wheat, but not connecting them somehow.

3. (18 points) Consider the LP: (It may be helpful to sketch it.) c_1 and c_2 are some constants.

$$\begin{array}{lll} \min & z = c_1x_1 + c_2x_2 & \\ \text{s.t.} & 3x_1 + 3x_2 & \geq 6 \\ & x_1 & \leq 5 \\ & x_1, x_2 & \geq 0 \end{array}$$

(a). Give an example for c_1, and c_2 such that the LP has a unique optimal solution. $c_1 = 2$, $c_2 = 1$, in other words $\min z = 2x_1 + x_2$ has a unique optimal solution.

Common mistake: giving an objective function that has multiple optimal solutions, such as $x_1 + x_2$ (or any objetive where $c_1 = c_2$).

(b). Give an example for c_1, and c_2 such that the LP is unbounded (optimal $z = -\infty$). $c_1 = 0$, $c_2 = -1$, in other words $\min z = -x_2$ is unbounded.

Common mistake: giving an objective function that has a bounded optimal solutions, such as $x_1 + x_2$ (or any objetive where $c_2 \geq 0$).

(c). Add a constraint to the problem so the LP has no feasible solutions. $x_1 + x_2 \leq 1$.

Common mistake: Adding $x_1 + x_2 \leq 2$ (or equivalently $3x_1 + 3x_2 \leq 6$), which imply that $x_1 + x_2 = 2$ but this IS feasible, for instance at $x_1 = 0$, $x_2 = 2$. Another mistake: adding a constraint with $<$ or $>$. These are not possible in Linear Programming. All inequalities should be $\leq$ or $\geq$.

4. (18 points) A company manufactures and sells dog food of two types. Each bag of type 1 dog food contains 2 pounds of lamb and 4 pounds of corn, sells for \$5. Each bag of type 2 dog food contains 1 pound of corn and 1 pound of lamb, sells for \$2. A total of 30 pounds of lamb and 50 pounds of corn are available. All dog food produced can be sold, and the company manager requires that at least 11 bags of dog food 1 are produced. Let $D1, D2$ be the number of bags of dog food type 1,2 produced. The following LP was formulated and then solved using Lindo to maximize the company's revenue. Answer each of the following, or explain why you cannot give an answer without rerunning Lindo:

$$
\begin{array}{llll}
\text{max} & 5D1 + 2D2 & \\
\text{s.t.} & 2) & 2D1 + D2 & \leq 30 \\
 & 3) & 4D1 + D2 & \leq 50 \\
 & 4) & D1 & \geq 11
\end{array}
$$

objective function value 67.0000000

variable	value	reduced cost
$D1$	11.000000	.000000
$D2$	6.000000	.000000

row	slack or surplus	dual prices
2)	2.000000	0.000000
3)	.000000	2.000000
4)	.000000	−3.000000

Range in which basis remains unchanged :

OBJ coefficient ranges

variable	current coef	allowable increase	allowable decrease
$D1$	5.000000	3.000000	infinity
$D2$	2.000000	infinity	.750000

righthand side ranges

row	current RHS	allowable increase	allowable decrease
2	30.000000	infinity	2.000000
3	50.000000	2.000000	6.000000
4	11.000000	1.500000	1.000000

(a). If 40 pounds of corn were available (instead of 50), what would be the new optimal solution to the problem? Reduce b_2 by 10 units, the allowable decrease is 6, so outside range. A new BFS will be optimal.
(b). What is the most that the company should be willing to pay to for another pound of corn? Dual price of corn constraint, \$2.
(c). The company just learned that each bag of dog food 1 can sell for \$6 (instead of \$5). What would be the new optimal solution to the problem (the z)? Increase is 1, allowable increase is 3, so within range. New $z = 67 + 11 = 78$.
5. (20 points) We wish to solve the LP below using the big M method.

$$
\begin{array}{lll}
\min & z = 2x_1 + x_2 & \\
\text{s.t.} & 3x_1 + 3x_2 & \geq 6 \\
 & x_1 & \leq 5 \\
 & x_1, x_2 & \geq 0
\end{array}
$$

(a). Rewrite the LP in standard form.

$$
\begin{array}{lll}
\min & z = 2x_1 + x_2 & \\
\text{s.t.} & 3x_1 + 3x_2 - e_1 & = 6 \\
 & x_1 + s_2 & = 5 \\
 & x_1, x_2, e_1, s_2 & \geq 0
\end{array}
$$

(b). Add artificial variable(s) and state the first tableau that will be used by the big M method (after "clean-up", eliminating the basic variables from the objective function)

Before "clean-up":

z	x_1	x_2	e_1	a_1	s_2	RHS
1	-2	-1	0	-M	0	0
0	3	3	-1	1	0	6
0	1	0	0	0	1	5

After "clean-up":

z	x_1	x_2	e_1	a_1	s_2	RHS
1	3M-2	3M-1	-M	0	0	6M
0	3	3	-1	1	0	6
0	1	0	0	0	1	5

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