AMS 341 (Spring, 2007) Estie Arkin

Operations Research I: Deterministic Models

Exam 2: Thursday, May 3, 2007

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 7 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. (Use back of pages also for scratch paper if you need it.)

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

Last (Family) Name (PRINT CLEARLY): ________________________

First Name (PRINT CLEARLY): ________________________

ID Number: ________________________

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I cheat I will be subjected to the maximum possible penalty permitted under University guidelines, including a "Q" grade for the course, an academic dishonesty notation on my transcript and even dismissal from the University.

Signature:

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1. (15 points) A developer is coordinating the construction of an office complex. The following activities would have to be undertaken before construction can begin:

Activity	Predecessors	Time (months)
A	-	4
B	-	6
C	A	2
D	A	6
E	B,C	3
F	B,C	3
G	D,E	5

(a). Draw a project network.

(b). What is the critical path for this project? You may find the path either by computing the early and late times for each node, or by inspection. Your answer should be a list all critical activities.

2. (18 points) Ms. Eff, the 6th grade science teacher is getting ready for the science fair. To do so, she must put each of the 8 student power point presentations on one of 2 USB drives, of capacity 128MB. The size of each presentation and its general topic are given in the table below. P is a physics project, C is a chemistry project, G is a geology project which is neither physics nor chemistry. The 8th project is both physics and chemistry. The assignment of projects to USB drives must satisfy the following conditions:

1. USB drive 2 must have exactly two physics projects.
2. USB drive 1 must have at least three chemistry projects.
3. Either project 5 or 6 (or both) must be on USB drive 1.
4. The total size of projects on USB drive 1 must be at least as large as the total size of projects on USB drive 2.

Presentation number	1	2	3	4	5	6	7	8
topic	P	C	P	C	P	C	G	P and C
size (MB)	34	45	30	20	26	32	28	38

Help Ms. Eff minimize the total size of projects on USB 1 by formulating an integer programming problem. (Do NOT solve - just formulate!)

(a). Define the variables:

(b). What is the objective function? (Max or Min?)

(c). What are the constraints?

3. (17 points) A politician is making plans for the upcoming elections. She has three volunteer workers to assign to its 3 precincts. Each volunteer can be assigned to exactly one precinct. The estimated increase in the number of votes for the party's candidate in each precinct if it were allocated various numbers of volunteers is given in the table below.

	0 volunteerss	1 volunteer	2 volunteers	3 volunteers
Precinct 1	0	4	9	15
Precinct 2	0	7	11	16
Precinct 3	0	5	10	15

The politician wishes to maximize the total estimated increase in the number of votes. Use dynamic programming to determine how many volunteers should be assigned to each precinct. To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum number of votes in stages i and above and state s.

Solve the problem. Make sure to state at the end how many volunteers are assigned to each precinct. (A solution by guessing will get no credit, I want to see your computations using $f_i(s)$ with the stages and states you defined.)

4. (10 points) We wish to solve an integer programming problem. All variables are restricted to be integer. We began by solving the LP relaxation of the problem and got the final (optimal) tableau for it. Unfortunately, not all the variables are integer.

z	x_1	x_2	x_3	x_4	x_5	RHS
1	0	-2	-3.5	0	0	2.35
0	0	2	0	1	0	10
0	0	0.6	-1.3	0	1	5.5
0	1	0	2	0	0	5

To solve the problem using the cutting plane method, what cut (constraint) would you add? Note: Do not solve the problem, just state the added constraint.

5. (10 points) Consider the following (minimum) Balanced Transportation problem: Find an initial BFS for the problem using the min cost method:

12	5	3	9	100
6	2	11	8	100
4	11	10	3	100
80	40	80	100	

6. (15 points) A real estate firm is considering the purchase of three buildings. The buildings costs 2, 5, and 6 million dollars respectively. The company has approached 4 banks for financing. The loan officers at the banks asses the risks, and each offers a loan of up to 4 million dollars at the following interest rates. A "-" means that a bank refuses to loan money for this building purchase.

	building 1	building 2	building 3
bank 1	-	15	13
bank 2	12	11	13
bank 3	10	15	17
bank 4	12	-	11

The company's goal is to minimize the cost of interest it pays on its loans. Formulate the problem as a Balanced Transportation problem by giving the the transportation tableau (cost and requirement matrix).

7. (15 points) Consider the following Linear Programming problem:

$$\begin{aligned} \max \quad & z = -4x_1 - x_2 \\ \text{s.t.} \quad & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 3 \\ & 3x_1 + x_2 = 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(a). What is the dual of the LP?

(b). The final tableau for the given LP is given below. e_1 is the excess variable of the first constraint, s_2 the slack variable of the second constraint, and a_1, a_3 the artificial variables of constraints 1,3. The final tableau was found using the big M method. What is the optimal solution to the dual? Make sure to state the objective value and the value of all dual variables.

z	x_1	x_2	e_1	s_2	a_1	a_2	RHS
1	0	0	0	1/5	M	M-7/5	-18/5
0	0	1	0	3/5	0	-1/5	6/5
0	1	0	0	-1/5	0	2/5	3/5
0	0	0	1	1	-1	1	0