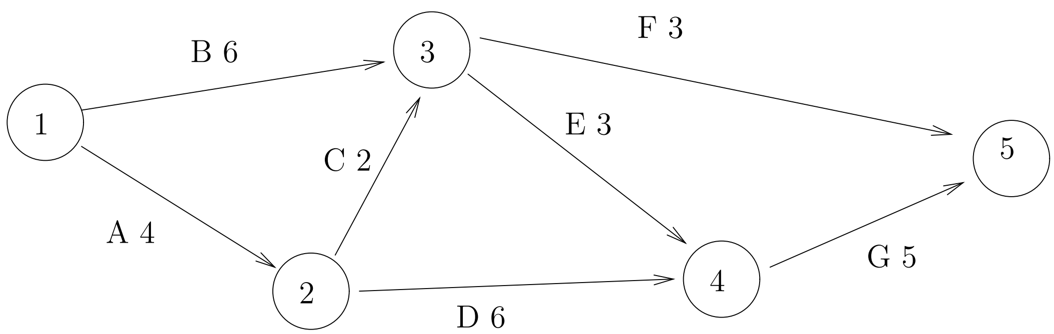
Mean 74, median 73.5, high 100, low 29.

1. (15 points) A developer is coordinating the construction of an office complex. The following activities would have to be undertaken before construction can begin:

Activity	Predecessors	Time (months)
A	-	4
B	-	6
C	A	2
D	A	6
E	B,C	3
F	B,C	3
G	D,E	5

(a). Draw a project network.



Common mistakes: Not having a single start node or single end node (F not connected to the end node), not numbering the nodes, making activities into nodes instead of arcs, not giving directions to arcs.

(b). What is the critical path for this project? You may find the path either by computing the early and late times for each node, or by inspection. Your answer should be a list all critical activities. A,D,G are critical. The length of the critical path is 15.

2. (18 points) Ms. Eff, the 6th grade science teacher is getting ready for the science fair. To do so, she must put each of the 8 student power point presentations on one of 2 USB drives, of capacity 128MB. The size of each presentation and its general topic are given in the table below. P is a physics project, C is a chemistry project, G is a geology project which is neither physics nor chemistry. The 8th project is both physics and chemistry. The assignment of projects to USB drives must satisfy the following conditions:

1. USB drive 2 must have exactly two physics projects.
2. USB drive 1 must have at least three chemistry projects.
3. Either project 5 or 6 (or both) must be on USB drive 1.
4. The total size of projects on USB drive 1 must be at least as large as the total size of projects on USB drive 2.

Presentation number	1	2	3	4	5	6	7	8
topic	P	C	P	C	P	C	G	P and C
size (MB)	34	45	30	20	26	32	28	38

Help Ms. Eff minimize the total size of projects on USB 1 by formulating an integer programming problem. (Do NOT solve - just formulate!)

(a). Define the variables: $x_i = 1$ if project i is on drive 1, and $x_i = 0$ otherwise. Similarly, $y_i = 1$ if project i is on drive 2, $y_i = 0$ otherwise. Note that $x_i + y_i = 1$ by definition, so we could use only the x_i variables.

(b). What is the objective function? (Max or Min?) $\min z = 34x_1 + 45x_2 + 30x_3 + 20x_4 + 26x_5 + 32x_6 + 28x_7 + 38x_8$

(c). What are the constraints?

$$\begin{aligned}
x_i + y_i &= 1,\ i = 1, ..., 8 \\
y_1 + y_3 + y_5 + y_8 &= 2 \\
x_2 + x_4 + x_6 + x_8 &\geq 3 \\
x_5 + x_6 &\geq 1 \\
34x_1 + 45x_2 + 30x_3 + 20x_4 + 26x_5 + 32x_6 + 28x_7 + 38x_8 &\geq \\
34y_1 + 45y_2 + 30y_3 + 20y_4 + 26y_5 + 32y_6 + 28y_7 + 38y_8 & \\
x_i,\ y_i \quad \text{binary} & \quad i = 1, ..., 8
\end{aligned}$$

Common mistakes: forgetting constraints such as each drive can contain at mose 128MB, additional variables that are not connected with the other variables (and may result in an optimal solution where all variables are 0), forgetting to say variables are binary (or integer), multiplying variables.

3. (17 points) A politician is making plans for the upcoming elections. She has three volunteer workers to assign to its 3 precincts. Each volunteer can be assigned to exactly one precinct. The estimated increase in the number of votes for the party's candidate in each precinct if it were allocated various numbers of volunteers is given in the table below.

	0 volunteers	1 volunteer	2 volunteers	3 volunteers	4 volunteers
Precinct 1	0	4	9	15	18
Precinct 2	0	7	11	16	18
Precinct 3	0	5	10	15	18

The politician wishes to maximize the total estimated increase in the number of votes. Use dynamic programming to determine how many volunteers should be assigned to each precinct. To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum number of votes in stages i and above and state s.

Solve the problem. Make sure to state at the end how many volunteers are assigned to each precinct. (A solution by guessing will get no credit, I want to see your computations using $f_i(s)$ with the stages and states you defined.)

Stages are precincts, states number of volunteers remaining to assign, $f_i(s)$ = max votes earned in precincts $i, ..., 3$

$f_3(0) = 0$, $f_3(1) = 5$, $f_3(2) = 10$, $f_3(3) = 15$, $f_2(0) = 0$, $f_2(1) = 7$, $f_2(2) = 12$, $f_2(3) = 17$, $f_1(3) = 17$, 0 volunteers to precinct 1, 1 volunteer to precinct 2, and 2 volunteers to precinct 3.

Note: a solution with no work (or incorrect work) got little partial credit!

4. (10 points) We wish to solve an integer programming problem. All variables are restricted to be integer. We began by solving the LP relaxation of the problem and got the final (optimal) tableau for it. Unfortunately, not all the variables are integer.

z	x_1	x_2	x_3	x_4	x_5	RHS
1	0	-2	-3.5	0	0	2.35
0	0	2	0	1	0	10
0	0	0.6	-1.3	0	1	5.5
0	1	0	2	0	0	5

To solve the problem using the cutting plane method, what cut (constraint) would you add? Note: Do not solve the problem, just state the added constraint.

Using the second constraint, we have $0.6x_2 - 1.3x_3 + x_5 = 5.5$, which is equivalent to $0x_2 - 2x_3 + x_5 - 5 = 0.5 - 0.6x_2 - 0.7x_3$, so the cut is $0.5 - 0.6x_2 - 0.7x_3 \leq 0$.

5. (10 points) Consider the following (minimum) Balanced Transportation problem: Find an initial BFS for the problem using the min cost method:

12 20	5	3 80	9	100
6 60	2 40	11	8	100
4 0	11	10	3 100	100
80	40	80	100	

Common mistakes: Missing a basic variable that is equal to 0 (there should be $3 + 4 - 1 = 6$ basic variables), Giving a feasible solution found by some other (unkown to me) method.

6. (15 points) A real estate firm is considering the purchase of three buildings. The buildings costs 2, 5, and 6 million dollars respectively. The company has approached 4 banks for financing. The loan officers at the banks asses the risks, and each offers a loan of up to 4 million dollars at the following interest rates. A "-" means that a bank refuses to loan money for this building purchase.

	building 1	building 2	building 3
bank 1	-	15	13
bank 2	12	11	13
bank 3	10	15	17
bank 4	12	-	11

The company's goal is to minimize the cost of interest it pays on its loans. Formulate the problem as a Balanced Transportation problem by giving the the transportation tableau (cost and requirement matrix).

	building 1	building 2	building 3	dummy	supply
bank 1	M	0.15	0.13	0	4
bank 2	0.12	0.11	0.13	0	4
bank 3	0.10	0.15	0.17	0	4
bank 4	0.12	M	0.11	0	4
demand	2	5	6	3	

Common mistakes: Problem not balanced (no dummy), leaving "-" instead of some cost, confusing with an assignment problem, costs for a full loan (such as 4 times 0.12) instead of per $.

7. (15 points) Consider the following Linear Programming problem:

$$\begin{array}{lll} \max & z = -4x_1 - x_2 & \\ \text{s.t.} & 4x_1 + 3x_2 & \geq 6 \\ & x_1 + 2x_2 & \leq 3 \\ & 3x_1 + x_2 & = 3 \\ & x_1, x_2 & \geq 0 \end{array}$$

(a). What is the dual of the LP?

$$\begin{array}{lll} \min & w = 6y_1 + 3y_2 + 3y_3 & \\ \text{s.t.} & 4y_1 + y_2 + 3y_3 & \geq -4 \\ & 3y_1 + 2y_2 + y_3 & \geq -1 \\ & y_1 \leq 0, \quad y_2 \geq 0 & y_3 \text{ unrestricted} \end{array}$$

(b). The final tableau for the given LP is given below. e_1 is the excess variable of the first constraint, s_2 the slack variable of the second constraint, and a_1, a_3 the artificial variables of constraints 1,3. The final tableau was found using the big M method. What is the optimal solution to the dual? Make sure to state the objective value and the value of all dual variables.

z	x_1	x_2	e_1	s_2	a_1	a_2	RHS
1	0	0	0	1/5	M	M-7/5	-18/5
0	0	1	0	3/5	0	-1/5	6/5
0	1	0	0	-1/5	0	2/5	3/5
0	0	0	1	1	-1	1	0

Following page 344 in text, $y_1 = 0$, $y_2 = 1/5$ $y_3 = M - 7/5 - M = -7/5$, $w = 6 \cdot 0 + 3(1/5) + 3(-7/5) = -18/5$