AMS 341 (Spring, 2008) Exam 2 - Solution notes Estie Arkin

Mean 85, median 86, high 100, low 46.

1. (15 points) In a large city a number of city workers will have their jobs eliminated because their skills are no longer needed. The three catagories to be eliminated include 20,15,25 people respectively. At the same time, the city finds itself needing an additional 30 electricians, 15 truck drivers and 10 computer operators. The city decided that instead of hiring new workers, it will retrain some of the workers whose jobs will be eliminated. The table below gives the retraining costs (in 100's of dollars) to move a worker from an obsolete catagory to one of the needed catagories. The cost of letting a worker go is 1100$.

	Electrician	Truck Driver	Computer Operator
catagory 1	8	6	4
catagory 2	5	4	5
catagory 3	6	7	9

The city's goal is to retrain and let go workers at minimum cost. Formulate the problem as a Balanced Transportation problem by giving the cost and requirement matrix.

	Electrician	Truck Driver	Computer Operator	unemplyed	supply
catagory 1	8	6	4	11	20
catagory 2	5	4	5	11	15
catagory 3	6	7	9	11	25
demand	30	15	10	5	

2. (10 points) We wish to solve an integer programming problem. All variables are restricted to be integer. We began by solving the LP relaxation of the problem and got the final (optimal) tableau for it. Unfortunately, not all the variables are integer.

z	x_1	x_2	x_3	x_4	x_5	RHS
1	0	-1.5	-3	0	0	6.25
0	0	3	0	1	0	10
0	1	2	5	0	0	5
0	0	1.2	-2.5	0	1	5.4

To solve the problem using branch and bound, what constraint would you add to get subproblem 2 and what constraint would you add to get subproblem 3 (from the LP relaxation which is subproblem 1)? Note: Do not solve the problem, just state the variable and the 2 constraints.

Branch on x_5 by adding the constraints $x_5 \leq 5$ to subproblem 2, and $x_5 \geq 6$ to subproblem 3.

3. (10 points) Consider the following (minimum) Balanced Transportation problem, and the given BFS.

	25	35	
20		10	
			30
		30	25

To get a better BFS, we decide to enter x_{31} into the basis. Find the next BFS given once x_{31} enters, and some other variable leaves the basis. Give your new BFS below (show only the basic variables):

	25	35	
		30	
20			10
		10	45

4. (15 points) Consider the following Linear Programming problem:

$$\begin{array}{lll} \max & z = 3x_1 + x_2 & \\ \text{s.t.} & 2x_1 + x_2 & \leq 4 \\ & 3x_1 + 2x_2 & \geq 6 \\ & 4x_1 + 2x_2 & = 7 \\ & x_1, x_2 & \geq 0 \end{array}$$

(a). What is the dual of the LP?

$$\min \quad w = 4y_1 + 6y_2 + 7y_3$$

$$\begin{array}{lll} \text{s.t.} & 2y_1 + 3y_2 + 4y_3 & \geq 3 \\ & y_1 + 2y_2 + 2y_3 & \geq 1 \\ & y_1 \geq 0, \ , y_2 \leq 0 & y_3 \text{ unrestricted} \end{array}$$

(b). The final tableau for the given LP is given below. e_2 is the excess variable of the second constraint, s_1 the slack variable of the first constraint, and a_2, a_3 the artificial variables of constraints 2,3. The final tableau was found using the big M method. What is the optimal solution to the dual? Make sure to state the objective value and the value of all dual variables.

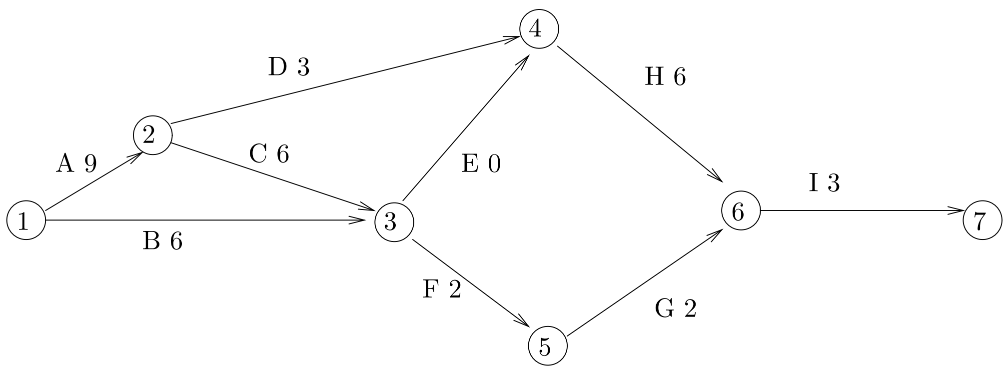
z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	0	0	0	1	M-1	M+3/2	9/2
0	0	0	1	0	0	-1/2	1/2
0	0	1	0	-2	2	-3/2	3/2
0	1	0	0	1	-1	1	1

Follow Page 310 in textbook: $y_1 = 0$, $y_2 = -1$, $y_3 = 3/2$, $w = 9/2$

5. (15 points) The County's Parks Commission is planning to develope a new park on a recently purchase 100-acre tract. Project developement activities include clearing playground area, constructing roads, purchasing picnic equipment and so on. The time (in weeks) and the predecessors of each activity are given in the table below:

Activity	A	B	C	D	E	F	G	H	I
Predecessors	-	-	A	A	B,C	B,C	F	D,E	G,H
Time (weeks)	9	6	6	3	0	3	2	6	3

(a). Draw a project network.



(b). What is the critical path for this project? You may find the path either by computing the early and late times for each node, or by inspection. Your answer should be a list all critical activities. Early event time:

Node	1	2	3	4	5	6	6
Early time	0	9	15	15	18	21	24
Late time	0	9	15	15	19	21	24

Total float: TF(1,2)=0, TF(2,3)=0, TF(3,4)=0, TF(4,6)=0, TF(2,4)=3, TF(1,3)=9, TF(3,6)=1, TF(5,6)=1, TF(6,7)=0

Critical path is activities A,C,E,H,I (or nodes 1-2-3-4-6-7) length of critical path 24 weeks.

(c). The park commissioner would like to open the park within 6 months of starting the project. Does this opening time appear to be feasible? Explain. Yes, 24 weeks is almost 6 months, so it is possible to complete the project withing 6 months.

6. (18 points) An emergency planning team for Long Island has identified five potential sites for basing paramedic teams, and has broken the region into six demand regions. The goal is to have paramedics based at most 5 minutes from the center of each demand region, and at least two bases at most ten minutes from the center of region 2 and the center of region 4, the areas of the most critical need. Travel time in minutes from the sites (identified by A,B,C,D,E) to the centers of the 6 regions are given below, as well as the costs of the proposed bases (in \$1000 units) are shown in the table below:

	site A	site B	site C	cite D	site E
region 1	5	14	7	4	8
region 2	11	4	6	5	12
region 3	7	6	4	15	5
region 4	8	15	5	6	4
region 5	4	9	11	5	8
region 6	1	6	15	10	3
Costs	100	90	85	110	95

Formulate an integer programming problem to minimize the cost of building the necessary bases. (Do NOT solve - just formulate!)

(a). Define the variables: a, b, c, d, e binary variables whether a base at that location is opened.

Common mistake, defining x_{ij} of regions and sites. Once a site is open, it services all regions that are close enough.

(b) (c).

$$
\begin{aligned}
\min \quad & z = 100a + 90b + 85c + 110d + 95e & \\
\text{s.t.} \quad & a + d & \geq 1 \\
& b + d & \geq 1 \\
& c + e & \geq 1 \\
& c + e & \geq 1 \\
& a + d & \geq 1 \\
& a + e & \geq 1 \\
& b + c + d & \geq 2 \\
& a + c + d + e & \geq 2 \\
& a, b, c, d, e & \text{binary}
\end{aligned}
$$

This is very similar to the covering problem done in class. A site "covers" a region, if it is close enough to that region. For example, region 1 can be covered by sites A or D, that are not more than 5 minutes away, but not by sites B,C,E, that are farther.

Common mistakes: confusing costs and distances.

7. (17 points) The AMS department has a total of 4 TAs available to TA classes next semester. From past experiance, the department knows that the average grade for a class will be higher, the more TAs are assigned to that class, given by the following table:

	0 TAs	1 TA	2 TAs	3 TAs	4 TAs
AMS 301	60	65	75	80	85
AMS 310	40	55	60	70	90
AMS 315	70	75	80	80	95

Assume that TAs cannot be split (meaning that each TA is assigned to a single course). The department wishes to maximize the sum of average scores in the three courses. To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum sum of average grades in stages i and above and state s.

$f_3(0) = 70$, $f_3(1) = 75$, $f_3(2) = 80$, $f_3(3) = 80$, $f_3(4) = 95$, $f_2(0) = 110$, $f_2(1) = 125$, $f_2(2) = 130$, $f_2(3) = 140$, $f_2(4) = 160$, $f_1(4) = 220$, assign all TAs to 310.