AMS 540 / MBA 540 (Fall, 2011) Estie Arkin

## Homework Set # 2

Due in class on Tuesday, September 20, 2011.

1). (a) Show that the set of all feasible solutions to the following linear program forms a convex set: $\min\{cx \mid Ax = b,\ x \geq 0\}$
(b) Prove that the set of optimal solutions to the linear program (of part (a)) forms a convex set.

2). An LP minimize $cx$ subject to $Ax \geq b$ and $x \geq 0$ was solved. Denote $x^*$ the optimal solution. Now a new constraint is added: $\alpha x \geq \beta$. Prove that if $x^*$ is feasible to the new constraint, then it is also an optimal solution to the new LP.

3). (From 2003 midterm) Consider a maximization linear programming problem with 4 extreme points $v_1, v_2, v_3, v_4$ and 3 extreme directions $d_1, d_2, d_3$, and with objective function coefficients given by the vector $c$, such that $cv_1 = 5$, $cv_2 = 18$, $cv_3 = 4$, $cv_4 = 18$, $cd_1 = -5$, $cd_2 = 0$, $cd_3 = -2$.
(a). Write down the reformulated LP in terms of the extreme points and extreme directions.
(b). Is the LP unbounded? Does it have a finite optimal solution? If so what is the optimal $z$? Does the LP have multiple optimal solutions? Explain! (Don't just say "yes" or "no".)

4). Consider the polyhedron $P = \{\mathbf{x} \mid \mathbf{x} \geq \mathbf{0}\}$.
(a). Show that $\mathbf{x} = \mathbf{0}$ is an extreme point of $P$.
(b). Show that any $\mathbf{x} \neq \mathbf{0}$, $\mathbf{x} \in P$ is not an extreme point. (Recall, $\mathbf{x} \neq \mathbf{0}$ means that at least one component of the vector $\mathbf{x}$ is not 0. Therefore $\mathbf{x} \neq \mathbf{0}$, and $\mathbf{x} \geq \mathbf{0}$ does *not* imply that $\mathbf{x} > \mathbf{0}$.)
(c). What are the extreme directions of $P$? Recall, an extreme direction of a set is a direction $\mathbf{d}$ that cannot be written as a *positive* combination of two distinct directions of the set, that is there do not exist directions $\mathbf{d_1}$ and $\mathbf{d_2}$ of the set, $\mathbf{d_1} \neq \mathbf{d_2}$, and $\alpha_1, \alpha_2 > 0$ such that $\mathbf{d} = \alpha_1 \mathbf{d_1} + \alpha_2 \mathbf{d_2}$.