

### Homework Set # 3

Due in class on Tuesday, October 4, 2011.

1). (Modified from an old midterm question) Consider the following LP, and its initial tableau  $x_5, x_6$  are the slack variables for the first and second constraints.

$$\begin{aligned} \max \quad z &= 30x_1 + 36x_2 - 3x_3 - 4x_4 \\ x_1 + x_2 - x_3 &\leq 5 \\ 5x_1 + 6x_2 - x_4 &\leq 10 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

$z$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	RHS
1	-30	-36	3	4	0	0	0
0	1	1	-1	0	1	0	5
0	5	6	0	-1	0	1	10

- (a). Which are the basic variables, and what is the initial  $B$ ?  
 (b). The initial tableau is not optimal. Which variable enters the basis, and which variable leaves the basis?  
 Now, consider one of the later tableaus obtained when simplex was applied to the problem:

$z$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	RHS
1	2	0	-9	0	12	4	100
0	1	0	-6	1	6	-1	20
0	1	1	-1	0	1	0	5

- (c). What is  $B$ , for the BFS in which  $x_4$  and  $x_2$  are the basic variables?  
 (d). What is  $B^{-1}$  at this BFS?  
 (e). Could this later tableau have been obtained from the initial tableau in one simplex pivot? Explain.  
 (f). The LP is unbounded (see  $x_3$ ). Give values of variables  $x_1, x_2, x_3, x_4$  that are feasible, and for which the objective function is equal to 1000.

2). Suppose we have a BFS that is nondegenerate. Further suppose that an improving nonbasic variable  $x_k$  enters the basis. Prove that if the minimum ratio test for choosing a leaving variable has a unique variable achieving the minimum,  $x_{B_r}$ , then the next BFS is also non-degenerate.

- 3). (a). Prove that when solving an LP using the Simplex method, a variable that leaves the basis at the  $k$ -th pivot, can not re-enter the basis at the  $(k + 1)$ -st pivot.  
 (b). Can a variable that entered the basis at the  $k$ -th pivot leave the basis at the  $(k + 1)$ -st pivot? If so, give a (small) example. If not, prove why not.  
 (c). Assume that the LP being solved is non-degenerate. Can a variable that leaves the basis at some pivot ever return to the basis? Explain briefly.