

### Homework Set # 4

Due in class on Tuesday, October 11, 2011.

**1).** Consider the big  $M$  method discussed in class, and the proof of case (2b). Here we show that it is important to examine the most positive  $z_j - c_j$ . Why? Well, problem  $P$  may be unbounded even if during the solution of the big  $M$  method on problem  $P(M)$  a non basic variable  $x_j$  wants to enter the basis, and  $y_j \leq 0$  and not all artificial variables are zero. To see an example, we will solve the following LP twice.

$$\begin{array}{ll} \min & z = -x_1 - x_2 \\ \text{s.t.} & x_1 - x_2 - x_3 = 1 \\ & -x_1 + x_2 + 2x_3 - x_4 = 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

- (a). Solve using the big  $M$  method, choose at each step the most positive  $z_j - c_j$  as your entering variable.  
 (b). Solve again, using the big  $M$  method, this time choose  $x_1$  as the entering variable in the first step, and see that  $P(M)$  is unbounded.

**2).** (from an old midterm) Consider the following LP:

$$\begin{array}{ll} \min & z = 2x_1 + 3x_2 \\ & 1/2x_1 + 1/4x_2 \leq 4 \\ & x_1 + 3x_2 \geq 20 \\ & x_1 + x_2 = 10 \\ & x_1, x_2 \geq 0 \end{array}$$

To solve the problem we first put it into standard form by adding a slack variable in the first row, and subtracting an excess variable in the second row.

(a). To solve the problem using the two phase approach, add artificial variables, and state the first tableau of phase I:

(b). The final tableau of phase I is given below. Note that some numbers are not known in the tableau. Fill in the missing numbers, what are  $w, \alpha_1, \alpha_2, \alpha_3$ ?

$z$	$x_1$	$x_2$	$s_1$	$e_2$	$a_2$	$a_3$	RHS
1	0	0	0	0	-1	-1	w
0	0	0	1	$\alpha_1$	1/8	-5/8	1/4
0	0	1	0	$\alpha_2$	1/2	-1/2	5
0	1	0	0	$\alpha_3$	-1/2	3/2	5

**3).** Consider an LP problem  $\{\min cx \mid Ax = b, x \geq 0\}$ , and the corresponding big M problem  $\{\min cx + \mathbf{1}Mx_a \mid Ax + Ix_a = b, x \geq 0, x_a \geq 0\}$ . Is it possible that the optimal solution value of the big M problem is unbounded and the optimal solution of the original problem is bounded? Explain.