AMS 540 / MBA 540 (Fall, 2011) Estie Arkin

Linear Programming

Homework Set # 6

Due in class on Thursday, November 3, 2011.

1). Consider the following Primal Linear Programming problem: (Here x, y, a, b, c are all vectors in $\Re^n$, x and y are the variables, a, b, c are arbitrary constant vectors.)

$$\begin{aligned} \min \quad & a^t x - b^t y \\ \text{s.t.} \quad & x - y \quad = c \\ & x \geq 0, \quad y \geq 0 \end{aligned}$$

(a). Exhibit a feasible solution to this Primal problem.
(b). Write the Dual of this Primal linear program.
(c). Since the Primal is feasible, either there exists an (bounded) optimal solution, or there exists a set of solutions with unbounded objective value. Derive the necessary and sufficient conditions on a and b for when the Primal has a bounded optimal solution. Hint: You can do this in one of two ways: Using the Dual problem, or by eliminating (i.e. subsituting for) y from the Primal.
(d). Assuming the Dual is feasible, exhibit an optimal solution to the Dual, and use it (and complementary slackness) to find an optimal solution to the Primal.

2). [IC 6.14] Suppose that the problem $\{\max cx \mid Ax = b,\ x \geq 0\}$ has a finite optimal solution. Let d be an arbitrary vector (of the same dimension as b). Show that if the problem $\{\max cx \mid Ax = d,\ x \geq 0\}$ has a feasible solution, then it has a finite optimal solution.

3). Use the dual simplex method to solve:

$$\begin{aligned} \max \quad & z = -2x_1 - x_3 \\ \text{s.t.} \quad & x_1 + x_2 - x_3 \geq 5 \\ & x_1 - 2x_2 + 4x_3 \geq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

4). A company manufactures 2 types of products. The only resource needed for the production is labour. Labourer 1 is willing to work upto 40 hours a week, and is paid \$5 an hour. Labourer 2 is willing to work upto 50 hours a week and is paid \$ 6 per hour. Product 1 sells for \$25, requires 1 hour of labourer 1 and 2 hours of labourer 2 and \$5 of raw material. Product 2 sells for \$22, requires 2 hours of labourer 1 and 1 hour of labourer 2 and \$4 of raw material. Let x_i be the number of product i made, we have the following LP:

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \leq 40 \\ & 2x_1 + x_2 \leq 50 \\ & x_1, x_2 \geq 0 \end{aligned}$$

The optimal solution to this LP $z = 80$, $x_1 = 20$ $x_2 = 10$.
(a). For what values of the price of type 1 product would the current BFS remain optimal?
(b). Graphically find the range of b_2 for which the current basis remains optimal and calculate the shadow price of the second constraint.