

## Linear Programming

### Homework Set # 8

Due in class on Tuesday, November 22, 2011.

1). Consider the following integer programming problem:

$$\begin{aligned}
 \max \quad & z = 4x_1 + 8x_2 \\
 \text{s.t.} \quad & 2x_1 + 2x_2 \leq 19 \\
 & -2x_1 + 2x_2 \leq 3 \\
 & x_1, x_2 \geq 0 \\
 & x_1, x_2 \text{ integer}
 \end{aligned}$$

The optimal tableau to the LP relaxation is:

	$z$	$x_1$	$x_2$	$x_3$	$x_4$	RHS
$z$	1	0	0	3	1	60
$x_1$	0	1	0	1/4	-1/4	4
$x_2$	0	0	1	1/4	1/4	11/2

(a). The optimum of the LP relaxation is  $x_1 = 4$ ,  $x_2 = 5.5$ . To solve the problem using the cutting plane method, what cut (constraint) would you add?

(b). Can the cutting plane method described in class be used to solve a mixed integer program (where specific variables are required to be integer, but not all variables). For instance, what if in the above problem only  $x_2$  is required to be integer, and  $x_1$  could be anything (non negative). Explain.

2). A company must meet the following demands for cash at the beginning of each of the next six months: Month 1 - \$200; month 2 - \$100; month 3 - \$50; month 4 - \$80; month 5 - \$160; month 6 - \$140. At the beginning of month 1, the company has \$150 in cash and \$200 worth of bond 1, \$100 worth of bond 2, and \$400 worth of bond 3. Of course the company will have to sell some bonds to meet demands, but a penalty will be charged for any bonds sold before the end of month 6. The penalties for selling \$1 worth of each bond are given in the table below. Assuming that all bills must be paid on time, formulate a balanced transportation problem that can be used to minimize the cost of meeting the cash demands for the next six months. Represent your formulation as a transportation tableau (cost and requirement table).

Bond	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
1	\$0.21	\$0.19	\$0.17	\$0.13	\$ 0.09	\$0.05
2	\$0.50	\$0.50	\$0.50	\$0.33	\$ 0	\$0
3	\$1.00	\$1.00	\$1.00	\$1.00	\$ 1.00	\$0