Important facts I should know (and remember)

Given two math programs $z_1 = \min\{f(x) \mid x \in P\}$, $z_2 = \min\{f(x) \mid x \in Q\}$, with $P \subseteq Q$ then $z_1 \geq z_2$.

If x_1 and x_2 are both feasible and optimal to an LP, then so is $\lambda x_1 + (1-\lambda)x_2$ for every $0 \leq \lambda \leq 1$. (See HW 2 problem 1.)

Consider an LP: $\min\{cx \mid Ax = b,\ x \geq 0\}$, A is an m by n matrix of linearly indep rows.

Definitions

- Basic Solution: Pick m cols of A that are *linearly indep*, call these cols B. Set the variables not corresponding to these col $x_N = 0$ and solve for the remaining (basic variables) $x_B = B^{-1}b$.
- Basic Feasible Solution (BFS): Basic Solution in which $x_B \geq 0$.
- Degenerate BFS: BFS in which one or more basic variable(s) is $= 0$.

Some important theorems

- If $\exists$ a finite optimal solution to the LP then $\exists$ an optimal BFS to the LP.
- x' optimal to LP does *not* necessarily imply x' is a BFS.
- number of BFS's $\leq$ number of BS's $\leq \binom{n}{m} < \infty$.
- Simplex applied to an LP in which all BFS are non degenerate *implies* BFS are not repeated *implies* Simplex will terminate in a finite number of steps.
- Simplex applied to an LP in which some BFS are degenerate *may* result in a BFS being repeated (cycling).