AMS 540 / MBA 540 (Fall, 2006) Estie Arkin

Linear Programming - Midterm

Do all problems. Write your answers on the exam. You are permitted to use the text, your notes and any material handed out in class. The exam time is 1 hour and 20 minutes. GOOD LUCK!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either receiving or giving unauthorized aid) I will get the grade "Q" for this course.

Name (PRINT CLEARLY), ID number and Signature:

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1). (25 points) Each day Eastinghouse produces capacitors during three shifts: 8am-4pm, 4pm-midnight, midnight-8am. The hourly salary paid to the employees of each shift, the price charged for each capacitor made during each shift, and the number of defects in each capacitor produced during a shift are given in the table below. Each of the company's 25 workers can be assigned to one of the three shifts. A worker produces 10 capacitors during a shift, but because of machinary limitations, no more than 10 workers can be assigned to each shift. Each day, at most 250 capacitors can be sold, and the average number of defects per capacitor for the day's production cannot exceed three. Formulate an LP to maximize Eastinghouse's daily profit (sales revenue minus labour cost). Make sure to clearly define your variables!

shift	hourly salary	defects (per capacitor)	price
8am-4pm	\$12	4	\$18
4pm-midnight	\$16	3	\$22
midnight-8am	\$20	2	\$24

2). (35 points) Consider the following simplex tableau for a minimization problem, in which the constraints are of the $\leq$ type, and s_i is the slack variable of constraint i. Note that some entries are missing:

z	x_1	x_2	s_1	s_2	s_3	RHS
1	0	α	0	β	0	f
0	1	-2	0	1	0	γ
0	0	-1	1	2	0	δ
0	0	0	0	3	1	e

Suppose that $\alpha < 0$, $\beta \leq 0$, and $\gamma, \delta, e \geq 0$
(a) What is the current B^{-1}?

(b) Is the tableau optimal?

(c) Give the original tableau (in terms of the unkowns).

Now suppose that $\alpha > 0$, $\beta \leq 0$, and $\gamma, \delta, e \geq 0$
(d) Is the tableau optimal?

(e) Let $\alpha = 3$ and $f = -8$. Give a feasible solution with $z = -200$.

3). (15 points) Consider the LP $\min\{cx \mid Ax \geq b,\ x \geq 0\}$. Let x^* be an optimal soltuion. Suppose that A is decomposed into $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ and b is decomposed into $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ such that $A_1x^* = b_1$ and $A_2x^* > b_2$.
Show (prove) that x^* is also an optimal solution to the problem LP_1: $\min\{cx \mid A_1x \geq b_1,\ x \geq 0\}$.

4). (25 points) (a). State a non degenerate BFS for the following LP: (make sure to say which variables are basic, which are non basic and what each variable is equal to.)

$$\begin{aligned} \max \quad & z = 6x_1 + 4x_2 + x_3 \\ \text{s. t.} \quad & x_1 + x_2 + x_3 \leq 1 \\ & x_1 \qquad\qquad \leq 1 \\ & x_1,\ x_2,\ x_3 \quad \geq 0 \end{aligned}$$

(b). State a degenerate BFS for the LP in part (a).

(c). Prove that Simplex will never cycle if an LP has the property that for every degenerate BFS the winner of the min ratio test is unique.