AMS 540 / MBA 540 (Fall, 2007) Estie Arkin

Linear Programming - Midterm

Do all problems. Write your answers on the exam. You are permitted to use the text, your notes and any material handed out in class. The exam time is 1 hour and 20 minutes. GOOD LUCK!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

Signature:

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Name (PRINT CLEARLY) and ID number:

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1). (18 points) The following is the first tableau of phase I for an LP, the artificial variables a_1, a_2, a_3 were added to constraints 1,2,3 respectively. Let $a_1 = x_7$, $a_2 = x_8$, $a_3 = x_9$.

w	x_1	x_2	x_3	x_4	x_5	x_6	a_1	a_2	a_3	RHS
1	0	0	1	0	2	-1	0	0	0	3
0	1	-1	1	0	2	0	1	0	0	0
0	-2	1	0	0	-2	0	0	1	0	3
0	1	0	1	0	1	-1	0	0	1	0
0	0	2	1	1	2	1	0	0	0	4

(a). Using Bland's rule, which variable enters the basis, and which variable leaves the basis?

(b). Using the lexicographic rule, which variable enters the basis, and which variable leaves the basis?

(c) Explain why phase I will usually have multiple (alternative) optimal solutions.

2). (25 points) During the next 2 months General Cars must meet (on time) the following demands for trucks and cars; month 1 - 300 trucks, 600 cars; month 2 - 300 trucks, 300 cars. During each month at most 1000 vehicles can be produced. Each truck uses 2 tons of steel, and each car uses 1 ton of steel. During month 1, steel costs $400 per ton; during month 2 steel costs $600 per ton. At most 1500 tons of steel may be purchased each month. Steel may only be used during the month it is purchased. At the end of each month a holding cost of $150 per vehicle is assessed. Each car gets 20 mpg, and each truck gets 10 mpg. During each month, the vehicles produced by the company must average at least 16 mpg. Formulate an LP to meet the demand at minimum cost. Make sure to clearly define your variables!

3). (30 points) Consider an LP: (p_1 and p_2 are given constants.)

$$\begin{aligned} \max \quad & z = -p_1 x_1 + p_2 x_2 \\ & x_1 - x_2 = 0 \\ & 0 \le x_1, x_2 \le 1 \end{aligned}$$

(a). What are the extreme points and extreme directions of this LP?

(b). Write the equivalent LP in terms of the extreme points and extreme directions.

(c). What is the dual of the (original) LP?

(d). Now suppose that $p_1 > p_2$. What is the optimal solution to the LP? (Make sure to state z as well as x_1, x_2.)

(e). What is the optimal solution to the dual LP (that you gave in part (c))? Make sure to state the optimal objective function as well as the value of the dual variables.

4). (15 points) Consider the LP (P) $\min\{cx \mid Ax = b,\ x \geq 0\}$. (As usual, assume that A is an m by n matrix and b is a vector of dim m.) Let B be a basis for an optimal soltuion. Now consider a new LP, (P(λ)) $\min\{cx \mid Ax = b + \lambda d,\ x \geq 0\}$, where λ is a scalar and d is a fixed non zero vector of dimension m, in other words, replace b by $b + \lambda d$.

(a). Prove that if B is an optimal (and feasible) basis for the original problem (i.e., when $\lambda = 0$) and also optimal (and feasible) for some $\lambda' > 0$ then B is an optimal and feasible basis for all $0 \leq \lambda \leq \lambda'$.

(b). Give a condition such that the basis B will be optimal and feasible for all $\lambda \geq 0$.

5). (12 points) Consider an LP $\min\{cx \mid Ax = b,\ e^t x = 1,\ x \geq 0\}$, where $e \in R^n$ is a vector of ones. Suppose we are given a tableau of a BFS for the LP, x', which objective value z_0. Since I am lazy, I do not wish to continue pivoting, but I would like to know how far my current solution z_0 can be from the optimal solution z^*. Give a lower bound on z^* in terms of the data in the current tableau. (In other words, your lower bound may only use the terms $\bar{b}$, y_{ik}, $z_j - c_j$, z_0, R.)