AMS 540 / MBA 540 (Fall, 2008) Estie Arkin

Linear Programming - Midterm

Do all problems. Write your answers on the exam. You are permitted to use the text, your notes and any material handed out in class. The exam time is 1 hour and 20 minutes. GOOD LUCK!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

Signature:

____

Name (PRINT CLEARLY) and ID number:

____

1). (30 points) Consider the standard form polyhedron $P = \{x \mid Ax = b,\ x \geq 0\}$. (As usual, assume that A is an m by n matrix whose rows are linearly independent, and b is a vector of dim m.) For each of the following statements, state whether it is true or false. If true, give a short proof, otherwise provide a counterexample or explanation.

(a). If there is more than one optimal solution, then there are infinitly many optimal solutions.

(b). If there is more than one optimal solution, then there are at least two Basic Feasible solutions that are optimal.

(c). At every optimal solution, no more than m variables can be positive.

(d). Let x' be a feasible solution with exactly m variables that are positive, then x' is a BFS.

(e). If the LP is degenerate, it may have an infinite number of BFS.

2). (30 points) Juiceco manufactures two products: premium orange juice and regular orange juice. Both products are made by combining two types of oranges: grade 6 and grade 3. The oranges in premium orange juice must have an average grade of at least 5, those in regular juice, at least 4. During each of the next two months Juiceco can sell upto 1,000 gallons of premium juice and upto 2,000 gallons of regular juice. Premium juice sells for $1.00 per gallon, while regular juice sells for 80 cents per gallon. At the beginning of month 1, Juiceco has 3,000 gallons of grade 6 oranges and 2,000 gallons of grade 3 oranges. At the beginning of month 2 Juiceco may purchase addition grade 3 oranges for 40 cents per gallon, and additional grade 6 oranges for 60 cents per gallon. Juice spoils at the end of the month so it makes no sense to make juice in month 1 to sell in month 2. Oranges left at the end of month 1 may be used to produce juice during month 2. At the end of month 1 a holding (inventory) cost of 5 cents is assessed against each gallon of leftover grade 3 oranges, and 10 cents against each gallon of grade 6 oranges. In addition to the cost of oranges, it costs 10 cents to produce each gallong of juice (regular or premium). Formulate an LP to maximize profit (revenues-costs) earned by Juiceco during the next two months. Make sure to clearly define your variables!

3). (20 points) Consider the LP $\min\{cx \mid Ax = b,\ x \geq 0\}$, and let $\min\{w = \mathbf{1}x_a \mid Ax + Ix_a = b,\ x, x_a \geq 0\}$ be the phase I problem ($\mathbf{1}$ is a vector of ones, x_a is a vector of artificial variables). In class we said that when solving the phase I problem, we can discard any artificial variable from the tableau whenever it leaves the basis. Let w^1 be the value of w at the termination of phase I when artificial variables are discarded (dropped) any time they leave the basis.

Now consider solving the phase I LP without discarding any variables, and let w^2 be the optimal value of the objective function.

(a). Prove that $w^1 \geq w^2$.

(b). Prove that if $w^1 > 0$ then $w^2 > 0$.

(c). Prove that $w^1 = 0$ if and only if $w^2 = 0$.

(d). Is w^1 always equal w^2? Give a proof or a brief explanation why not.

4). (20 points) Consider an LP in standard form $\min\{cx \mid Ax = b,\ x \geq 0\}$. (As usual, assume that A is an m by n matrix whose rows are linearly independent, and b is a vector of dim m.)

Suppose you know that every Basic Feasible Solution has at most one basic variable that is equal to zero (all BFS are "almost non degenerate").

(a). Prove that the Simplex method either terminates or strictly improves the objective function value after at most n iterations, given a starting Basic Feasable Solution.

(b). Prove that in this case Simplex will terminate in a finite number of steps. (Do not use any of the methods to avoid cycling, just prove this based on part (a).)