AMS 540 / MBA 540 (Fall, 2009) Estie Arkin

Linear Programming - Midterm

Do all problems. Write your answers on the exam. You are permitted to use the text, your notes and any material handed out in class. The exam time is 1 hour and 20 minutes. GOOD LUCK!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

Signature:

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Name (PRINT CLEARLY) and ID number:

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1). (20 points) Consider the following LP, in which c_1, c_2 are some constants:

$$\begin{array}{lll} \max & z = c_1x_1 + c_2x_2 & \\ \text{s.t.} & x_1 - 2x_2 & \leq 8 \\ & x_1 & \geq 2 \\ & 2x_1 + 3x_2 & \geq 12 \\ & x_1, x_2 & \geq 0 \end{array}$$

(a). Find all extreme points of the feasible region.

(b). The (normalizied) extreme directions of the feasible region are $d_1 = (0, 1)$, and $d_2 = (2/3, 1/3)$. Write the equivalent LP in terms of the extreme points and extreme directions.

(c). Characterize all pairs of objective function coefficients (c_1, c_2) for which the LP has a finite optimal solution.

2). (30 points) A bank is open Monday-Friday from 9am to 5pm. From past experiance, the bank knows that it needs (at least) the following number of tellers:

Time period	9-10	10-11	11-noon	noon-1	1-2	2-3	3-4	4-5
Tellers required	4	3	4	6	5	6	8	8

The bank hires two types of tellers. Full time tellers work 9-5 every day, except for 1 hour off for lunch. (The bank determines when a full time teller takes lunch hour, but it must be either noon-1 or 1-2.) Full time employees are paid $8 per hour (this includes payment for the lunch hour). The bank can also hire part time tellers. Each part time teller must work exactly 3 consecutive hours each day, and gets paid $5 per hour. To maintain quality of service, at most 5 part time tellers can be hired. Formulate an LP to minimize the cost of the bank to meet teller requirements. Make sure to clearly define your variables!

3). (30 points) The following is a tableau of a minmization problem. State conditions on $a_1, a_2, b, c_1, c_5, c_6$ that are required to make the following true:

z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	c_1	0	0	0	c_5	c_6	0
0	-1	1	0	0	a_1	0	1
0	-1	0	0	1	-2	1	2
0	a_2	0	1	0	1	1	b

(a). The current solution is feasible and optimal.

(b). The current basic solution is not feasible.

(c) The current basic solution is degenerate.

(d) The current basic solution is feasible, and the LP is unbounded.

(e) The current basic solution is feasible, and using Bland's rule, the next entering variable is x_6 and the departing variable is x_3.

4). (20 points) Consider an LP (P) $\min\{cx \mid Ax = b, \;\; x \geq 0\}$, and its corresponding big M LP, $\{\min cx + \mathbf{1}Mx_a \mid Ax + Ix_a = b, \; x \geq 0, \; x_a \geq 0\}$, called P(M).
(a). Prove that if $M_1 \leq M_2$ then $z(M_1) \leq z(M_2)$.

(b). Suppose that (P) has a finite optimal solution and denote it z^*. Prove that $z(M) \leq z^*$.

(c). Show that there is a value M' such that for $M \geq M'$, $z(M) = z^*$, and so we can conclude that the big-M method will produce the right solution for large enough M. (For this part, suppose as in the previous part that (P) has a finite optimal solution z^*.)