

**Theorem.** *The following three assertions are equivalent.*

- (1) *The collection  $\{T^k \tilde{B}_j\}$  forms a basis of decomposition which are  $\varepsilon \pmod 0$ .*
- (2) *The  $S$ -code  $A = \{A_i\}$  covers the Bernoulli scheme  $(X_Z, \mu_p, T)$ , or does not cover it, but the alphabet  $Z$  contains a letter  $a_k$  such that almost every realization can be represented (uniquely up to an indexing  $j$ , a possible shift) in the form*

$$\cdots a_k B_{j-n} a_k \cdots a_k B_{j-n+1} a_k \cdots a_k B_{j_0} a_k \cdots B_{j_1} \cdots a_k B_{j_n} a_k \cdots,$$

where  $B_{j_l}$ ,  $-\infty < l < \infty$ , is a word of any kind; i.e. all "lacunae" between words of whatever kind (for some  $l$  there may be none) are filled by letters  $a_k$  in arbitrary number.

- (3) *For all letters  $a_j \in Z$ , except possibly one (the  $a_k$  in 2)),*

$$P(a_j) = \sum_i k_j^i t_0^{A_i l - 1} \mu_p(\tilde{A}_i),$$

where  $t_0$  is the smallest positive real zero of the polynomial (series)

$$f(t) = 1 - t + \sum \mu_p(A_i) t^{A_i}.$$

The equality  $t_0 = (1 - \sum \mu_p(\tilde{B}_j))^{-1}$  holds, in which the summation ranges over all words of any kind. There is coverability when all equalities in (3) are satisfied for all  $a_j$ . If  $k_j$  and  $q_j$  satisfy the conditions of Theorem 1 in [1], then we take the canonical  $S$ -code described on p. 396 in [1], and  $s$  to be the solution  $t_0$  (see above) of the polynomial (series)  $f(t) = 1 - t + \sum q_j t^{k_j}$ . The equalities in condition (3) of the above theorem are obviously satisfied for all  $a_j$ , except possibly  $a_0$ ; so that any collection  $k_j$  and  $q_j$  (p. 395) which can be realized on an  $S$ -code and Bernoulli scheme may be realized on a code and scheme for which conditions (1)–(3) of the theorem hold. It is sufficient therefore that there exist a positive root of  $f(t)$ , whereas coverability requires the existence of a multiple positive root, being thus a stronger condition.

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A. N. Livshits  
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#### REFERENCES

- [1] A. N. LIVSHITS, *On the isomorphism problem for Bernoulli schemes*, Theory Prob. Appl., 19 (1974), pp. 394–398.

Through our own fault, the following errors were committed in the report *On homogeneous controlled Markov models with continuous time and finite or countable number of states* (this Journal, 24 (1979), pp. 156–161).

(1) In the definition of the estimate of the strategy on p. 156, it is necessary to interpret  $\mathbf{P}_x^\pi$  as  $\mathbf{P}_{s,x}^\pi$ , the measure corresponding to the process emitted at the moment  $s$  from the state  $x$  under the strategy  $\pi$  reckoned from the moment  $s$ .

(2) In part A of Theorem 2, the words "the model is summable from above and  $v \neq \infty$ " should be replaced by "the model is summable and  $v > -\infty$ ".

A. A. Yushkevich, E. A. Fainberg  
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