AMS-511 Foundations of Quantitative Finance

Fall 2015 — Class 08 — Monday 19 Oct 2015

Mid-Term Examination

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Note: Partial credit will be given where appropriate but only if you briefly show your work or explain the reasoning behind your answer. Neatness counts. If I can't understand what you have written, then I can't take your reasoning into account—even if you come up later to explain it. Your answers must stand as originally presented. Be brief and don't overcomplicate but be clear.

1. Time Value of Money (20 pts.)

Assume that the current spot rates are:

<table>
<thead>
<tr>
<th>Years</th>
<th>Spot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>0.035</td>
</tr>
<tr>
<td>3</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Roberta borrows $10,000 from Marsha and agrees to pay her interest according to the spot rate curve at the origination of the loan, i.e., per the table above. Assume annual compounding throughout.

(a) At the end of the first year, Roberta pays Marsha $3,000. Taking accrued interest into account, after that payment what is the balance at the end of year 1 owed to Marsha?

(b) Given the payment made in (a) above, at the end of the third year, Roberta pays Marsha the remaining balance of the loan. Taking accrued interest into account, what is that final balance paid to Marsha?

(c) Now assume an alternate case: Roberta borrows the $10,000 from Marsha and then makes only a single payment at the end of the third year. Taking accrued interest into account, what would be the amount of that single payment?

(d) Assuming (c) above, what was the effective annual interest rate paid by Roberta to Marsha?

2. Capital Budgeting (20 pts.)

A firm is considering a number of capital projects. It has a total investment budget of \( k \). You are given a list of 6
projects with their costs $c = \{c_1, \ldots, c_6\}$ and present values $v = \{v_1, \ldots, v_6\}$. The simplest form of the capital budgeting problem can be modeled using the follow mathematical programming:

$$\min \left\{ v^T x \mid c^T x \leq k \land x_i \in [0, 1] \text{ where } i \in \{1, \ldots, 6\} \right\}$$

where $x_i = 0$ if the $i$th project is not selected and $x_i = 1$ if it is.

However, there are these additional complications that all must be considered. [Hint: Remember that the variables are already constrained such that $x_i \in \{0, 1\}, \forall i$]

(a) If project 1 is selected, then project 2 must also be selected; however, if project 2 is selected, then it is possible but not necessary to select project 1.

(b) Projects 5 and 6 are alternate bridge designs. Neither may be selected, but if one of them is selected then the other one must not be selected.

(c) Project 4 is a roadway modification that must be selected if one of the bridge projects is selected, i.e., if either project 5 or 6 is selected, and which must not be selected if there is no bridge project, i.e., if projects 5 and 6 are not selected. [Hint: Consider the effect on the sum $x_5 + x_6$ of the constraint imposed by (b).]

(d) Either both projects 2 and 3 must be selected or both projects 2 and 3 must not be selected.

For each of the cases above determine the constraints which must be added to the mathematical program to ensure a feasible solution is found.

3. Dividend Discount Models (15 pts.)

The stock XYZ is expected to pay an annual dividend of $4.50/share on its next dividend date in one year.

(a) Assuming an annual dividend growth rate of 8% and an interest rate of 10%, estimate the price of XYZ using a dividend discount model.

(b) All other factors held constant, what would the growth rate have to fall to in order to cut the price of the stock in half?

(c) The market price of XYZ is $300. Assuming the same $4.50 up-coming dividend and 10% interest rate, what is the implied growth rate $g$ of XYZ?

4. Portfolio Optimization (20 pts.)

Two uncorrelated risk assets have mean returns of 8% and 15%, respectively, and standard deviations of return of 8% and 20%, respectively. The two risk assets are uncorrelated. The risk-free rate of return is 0.5%. Assume that you have one unit of capital to allocate and that there is no restrictions on short selling:

(a) What allocation of the risk assets represents the minimum variance portfolio?

(b) What is the mean and standard deviation of the minimum variance portfolio?

(c) What allocation of the risk assets represents the tangent portfolio?

(d) What is the mean and standard deviation of the tangent portfolio?
5. **Mathematica (25 pts.)**

What is the result of executing the following in *Mathematica*? You can either write out the actual result or explain in words what the code does. In either case, however, you are more likely to get partial credit if you *briefly* explain your answer.

(a) \(\{1, 2, 3, 4\}^2/\{1, 2, 3, 4\}\)

(b) \(\text{Table}[\{\theta, \text{Sin}[\theta]\}, \{\theta, 0, 2\pi, 0.5\pi\}]\)

(c) Evaluate \(0.5 \times ((2, 4, 8, 16) - 12)\).

(d) Evaluate \(\{(0, 1), (3, 5)\}.\{1, 1\}\)

(e) Evaluate \(\#^2 - 1 \& @ \{1, 2, 3\}\)