Lecture 9 Summary (Chapter 4)

3) Relative frequency (Equally likely outcomes)

If there are n equally likely probabilities, of which one must occur and s are regarded as a success, then the probability of a success is given as s/n.

Example 5 Probability of drawing an ace from a well-shuffled deck of 52 playing cards: $s/n = 4/52 = 1/13$

Example 6 Tossing a fair die: $S = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$
Let $A = \{1,\ 3,\ 5\}$. Probability of obtaining an odd number is

$$\text{Probability of } A = \frac{\#\text{elements in } A}{\#\text{elements in } S} = \frac{3}{6} = \frac{1}{2}.$$

Example 7 Tossing a fair die twice:
Let $A = \{\text{sum of the two numbers is } 6\} = \{(1,5),\ (2,4),\ (3,3),\ (4,2),\ (5,1)\}$.

$$\text{Probability of } A = \frac{\#\text{elements in } A}{\#\text{elements in } S} = \frac{5}{36}$$

4) Probability

Probability of an event:

(i) $0 \le P(A) \le 1$ for all event A
(ii) $P(A) = \sum_{(\text{all } e \text{ in } A)} P(e)$
(iii) $P(S) = \sum_{(\text{all } e \text{ in } S)} P(e) = 1$

Example 8 Tossing a fair coin: $S = \{H,\ T\}$

$$P(H) = P(T) = \frac{1}{2} \qquad P(S) = P(H) + P(T) = 1$$

- **Law of Complement:** $P(\overline{A}) = 1 - P(A)$

Example 9 Tossing a fair coin twice: $S = \{HH,\ HT,\ TH,\ TT\}$

$A = \{\text{two heads}\} = \{HH\}$
$\overline{A} = \{\text{at least one is tail}\} = \{HT,\ TH,\ TT\}$

$$P(A) = \frac{1}{4}, \quad P(\overline{A}) = \frac{3}{4} = 1 - P(A)$$

- If A and B are mutually exclusive, then $P(AB) = 0$.
- $P(A \cup B) = P(A) + P(B) - P(AB)$
- If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

Example 10 60% of freshmen in a college take calculus, 75% take history and 50% take both calculus and history.

(a) Probability of taking at least one of these courses:
Let $A = \{\text{taking calculus}\}$ and $B = \{\text{taking history}\}$
$\Longrightarrow P(A) = .6,\ P(B) = .75$ and $P(AB) = .5$

$$P(A \cup B) = P(A) + P(B) - P(AB) = .6 + .75 - .5 = .85$$

(b) Probability of taking only one of these courses:

$$P(A \cup B) - P(AB) = .85 - .5 = .35$$