Inferences about Population Means

1. known σ. Large sample or normal population

Estimation

Parameter	Point Estimate	Confidence Interval
μ	$\overline{x}$	$\overline{x} \pm z_{\alpha/2}\sigma/\sqrt{n}$

Testing

	Case (a)	Case (b)	Case (c)
Step 1	$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$ (2-sided alternative)	$H_0 : \mu \leq \mu_0$ $H_1 : \mu > \mu_0$ (1-sided alternative)	$H_0 : \mu \geq \mu_0$ $H_1 : \mu < \mu_0$ (1-sided alternative)
Step 2	$\alpha =?$	$\alpha =?$	$\alpha =?$
Step 3	$z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$ Rejection region $\lvert z \rvert \geq z_{\alpha/2}$	$z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$ Rejection region $z \geq z_\alpha$	$z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$ Rejection region $z \leq -z_\alpha$
Step 4	Substitute $\overline{x}$, s and n $z =?$ Decision	$z =?$ Decision	$z =?$ Decision
Step 5	$p = 2\times$ area	$p =$ area	$p =$ area

Sample size determination

Sample size needed to attain maximum error of estimate E:

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$$

Two types of errors

		H_0 is true	H_1 is true
Decision	H_0	Correct Decision	Type II Error
	H_1	Type I Error	Correct Decision

$\alpha = \mathrm{P}(\text{Type I error})$
$\beta = \mathrm{P}(\text{Type II error})$
power $= 1 - \beta$

Meaning of the p-value

A small value for p implies that it would be very unlikely to obtain a value of the test-statistic such as the one we observed if H_0 actually were true. The smaller the value of p, therefore, the more contradictory the sample results are to H_0 (i.e., the stronger is the evidence for H_1).
Note: Reject H_0 if $p \leq \alpha$, Do not reject H_0 if $p > \alpha$

Relationship between CI and test

CI $\equiv$ the acceptance region in a two-sided test.
The parameter is included in the CI $\Longleftrightarrow$ Accept H_0
The parameter is NOT included in the CI $\Longleftrightarrow$ Reject H_0
in a two-sided test

2. **Unknown σ. Large sample or normal population**

Estimation

Parameter	Point Estimate	Confidence Interval
μ	$\overline{x}$	$\overline{x} \pm t_{\alpha/2,n-1} s/\sqrt{n}$

Testing

	Case (a)	Case (b)	Case (c)
Step 1	$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$ (2-sided alternative)	$H_0 : \mu \leq \mu_0$ $H_1 : \mu > \mu_0$ (1-sided alternative)	$H_0 : \mu \geq \mu_0$ $H_1 : \mu < \mu_0$ (1-sided alternative)
Step 2	$\alpha =?$	$\alpha =?$	$\alpha =?$
Step 3	$t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$ Rejection region $\lvert t\rvert \geq t_{\alpha/2,n-1}$	$t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$ Rejection region $t \geq t_{\alpha,n-1}$	$t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$ Rejection region $t \leq -t_{\alpha,n-1}$
Step 4	Substitute $\overline{x}$, s and n $t =?$ Decision	$t =?$ Decision	$t =?$ Decision
Step 5	$p = 2\times$ area	$p =$ area	$p =$ area