Inferences about Two Population Means

1. **Large independent samples (both $n_1, n_2 \geq 30$)**

Estimation

Parameter	Point Estimate	Confidence Interval
$\mu_1 - \mu_2$	$\overline{x} - \overline{y}$	$(\overline{x} - \overline{y}) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Testing

	Case (a)	Case (b)	Case (c)
Step 1	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_a : \mu_1 - \mu_2 \neq \Delta_0$	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_a : \mu_1 - \mu_2 > \Delta_0$	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_a : \mu_1 - \mu_2 < \Delta_0$
Step 2	$\alpha =?$	$\alpha =?$	$\alpha =?$
Step 3	$z = \frac{\overline{x}-\overline{y}-\Delta_0}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$ Rejection region $\lvert z\rvert \geq z_{\alpha/2}$	$z = \frac{\overline{x}-\overline{y}-\Delta_0}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$ Rejection region $z \geq z_\alpha$	$z = \frac{\overline{x}-\overline{y}-\Delta_0}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$ Rejection region $z \leq -z_\alpha$
Step 4	Substitute $\overline{x}$, $\overline{y}$, s_1^2, s_2^2, n_1 and n_2		
	$z =?$ Decision	$z =?$ Decision	$z =?$ Decision
Step 5	$p = 2\times$ area	$p =$ area	$p =$ area

2. **Two independent samples ($n_1 < 30$ or $n_2 < 30$), both populations are normal**

A. When we assume $\sigma_1 = \sigma_2 = \sigma$

Pooled variance: $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Estimation

Parameter	Point Estimate	Confidence Interval
$\mu_1 - \mu_2$	$(\overline{x} - \overline{y})$	$(\overline{x} - \overline{y}) \pm t_{\alpha/2,n_1+n_2-2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Testing

	Case (a)	Case (b)	Case (c)
Step 1	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_1 : \mu_1 - \mu_2 \neq \Delta_0$	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_1 : \mu_1 - \mu_2 > \Delta_0$	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_1 : \mu_1 - \mu_2 < \Delta_0$
Step 2	$\alpha =?$	$\alpha =?$	$\alpha =?$
Step 3	$t = \frac{\overline{x}-\overline{y}-\Delta_0}{s_p\sqrt{1/n_1+1/n_2}}$ Rejection region $\lvert t \rvert \geq t_{\alpha/2,n_1+n_2-2}$	$t = \frac{\overline{x}-\overline{y}-\Delta_0}{s_p\sqrt{1/n_1+1/n_2}}$ Rejection region $t \geq t_{\alpha,n_1+n_2-2}$	$t = \frac{\overline{x}-\overline{y}-\Delta_0}{s_p\sqrt{1/n_1+1/n_2}}$ Rejection region $t \leq -t_{\alpha,n_1+n_2-2}$
Step 4	Substitute $\overline{x}$, $\overline{y}$, s_1^2, s_2^2, n_1 and n_2		
	$t =?$ Decision Rejection region $\lvert t \rvert \geq t_{\alpha/2,n_1+n_2-2}$	$t =?$ Decision Rejection region $t \geq t_{\alpha,n_1+n_2-2}$	$t =?$ Decision Rejection region $t \leq -t_{\alpha,n_1+n_2-2}$
Step 5	$p = 2\times$ area	$p =$ area	$p =$ area

B. When we assume $\sigma_1 \neq \sigma_2$

Estimated degrees of freedom:

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

Round down to the nearest integer.

Estimation

Parameter	Point Estimate	Confidence Interval
$\mu_1 - \mu_2$	$\overline{x} - \overline{y}$	$(\overline{x} - \overline{y}) \pm t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Testing

	Case (a)	Case (b)	Case (c)
Step 1	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_1 : \mu_1 - \mu_2 \neq \Delta_0$	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_1 : \mu_1 - \mu_2 > \Delta_0$	$H_0 : \mu_1 - \mu_2 = \Delta_0$ $H_1 : \mu_1 - \mu_2 < \Delta_0$
Step 2	$\alpha =?$	$\alpha =?$	$\alpha =?$
Step 3	$t = \frac{\overline{x}-\overline{y}-\Delta_0}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$ Rejection region $\lvert t \rvert \geq t_{\alpha/2,\nu}$	$t = \frac{\overline{x}-\overline{y}-\Delta_0}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$ Rejection region $t \geq t_{\alpha,\nu}$	$t = \frac{\overline{x}-\overline{y}-\Delta_0}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$ Rejection region $t \leq -t_{\alpha,\nu}$
Step 4	Substitute $\overline{x}$, $\overline{y}$, s_1^2, s_2^2, n_1 and n_2		
	$t =?$ Decision	$t =?$ Decision	$t =?$ Decision
Step 5	$p = 2\times$ area	$p =$ area	$p =$ area

3. **Paired Data**

Instead of considering x and y, we consider the difference d. Hence we have only one sample of size n, sample mean $\overline{d}$ and sample standard deviation s_d.

Estimation

Parameter	Point Estimate		Confidence Interval
μ_d: difference in population means	$\overline{d}$	$d.f. = n-1$	$\overline{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$ Check $t-$table

Testing

	Case (a)	Case (b)	Case (c)
Step 1	$H_0 : \mu_d = 0$ $H_1 : \mu_d \neq 0$	$H_0 : \mu_d = 0$ $H_1 : \mu_d > 0$	$H_0 : \mu_d = 0$ $H_1 : \mu_d < 0$
Step 2	$\alpha =?$	$\alpha =?$	$\alpha =?$

Use t-table, df $= n-1$

Step 3	$t = \frac{\overline{d}}{s_d/\sqrt{n}}$ Rejection region $\lvert t\rvert \geq t_{\alpha/2}$	$t = \frac{\overline{d}}{s_d/\sqrt{n}}$ Rejection region $t \geq t_\alpha$	$t = \frac{\overline{d}}{s_d/\sqrt{n}}$ Rejection region $t \leq -t_\alpha$
Step 4	Substitute $\overline{d}$, s_d and n		
	$t =?$ Decision	$t =?$ Decision	$t =?$ Decision
Step 5	$p = 2\times$ area	$p =$ area	$p =$ area