2000 Presidential Election

1) Weak Law of Large Numbers (WLLN)

- If you toss a fair coin infinitely many times, the chance of getting the head converges to 0.5.
- Bush: 2,910,198 (50.00281%)
- Gore: 2,909,871 (49.99719%)
- Difference: 327 votes
- Experiment of tossing a fair coin 5,820,069 times
  - A run of a sample program: done in class
  - The difference is more than 327 most of the time.
  - What is the probability that the difference is no more than 327?
  - Answer: will be obtained after next two lectures.
1) The Binomial Distribution

The binomial distribution:

\[ n: \text{a fixed number of Bernoulli trials} \]
\[ p: \text{the probability of success in each trial} \]
\[ X: \text{number of successes in } n \text{ trials} \]

\[ X: \text{binomial random variable } \implies X \sim \text{Bin}(n, p) \]

**Example 1**  30% of the trees in a forest are infested with a parasite. Four trees are selected at random. Let \( X \) be the number of trees sampled that have the parasite. \( I \): infested, and \( N \): not infested.

\[
\begin{array}{cccccc}
X = 0 & X = 1 & X = 2 & X = 3 & X = 4 \\
NNNN & NNNI & NNII & NIIP & IIII \\
NNIN & NINI & INII & NIIIP & IIIP \\
NINN & NIIN & IINII & IIIP & IIIP \\
INNN & INII & IIINII & IIII & IIII \\
ININ & IIINII & IIII & IIII & IIII \\
IINN & IIII & IIII & IIII & IIII \\
\end{array}
\]

<table>
<thead>
<tr>
<th>Probability of each outcome</th>
<th>((1 - p)^4)</th>
<th>(p(1 - p)^3)</th>
<th>(p^2(1 - p)^2)</th>
<th>(p^3(1 - p))</th>
<th>(p^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#outcomes</td>
<td>(\binom{4}{0} = 1)</td>
<td>(\binom{4}{1} = 4)</td>
<td>(\binom{4}{2} = 6)</td>
<td>(\binom{4}{3} = 4)</td>
<td>(\binom{4}{4} = 1)</td>
</tr>
</tbody>
</table>

\[
f(x) = P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \ldots, n
\]
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\[ P(X = 3) = \binom{4}{3} = 0.0756 \]

\[ P(X \geq 3) = P(X = 3) + P(X = 4) = \binom{4}{3}(0.3)^3(0.7) + \binom{4}{4}(0.3)^4 = 0.0837 \]

\[ P(X \leq 1) = P(X = 0) + P(X = 1) = \binom{4}{0}(0.7)^4 + \binom{4}{1}(0.3)(0.7)^3 = 0.6517 \]

\[ P(X > 3) = P(X = 4) = \binom{4}{4}(0.3)^4 = 0.0081 \]

\[ P(X < 2) = P(X \leq 1) = 0.6517 \]

Binomial tables are available.
Example 2  If the probability is 0.05 that a certain wide-flange column will fail under a given axial load, what are the probabilities that among 16 such columns,

(a) at most two will fail?
   Since $X \sim \text{Bin}(16, .05)$, $P(X \leq 2) = .957$.
(b) at least four will fail?
   $P(X \geq 4) = 1 - P(X \leq 3) = 1 - .993 = .007$

\[
X \sim \text{Bin}(n, p) \implies \mu = np, \quad \sigma^2 = np(1 - p), \quad \sigma = \sqrt{np(1 - p)}
\]

Example 3  Find the mean and variance of the probability distribution of the number of heads obtained in three flips of a balanced coin.

$X \sim \text{Bin}(3, .5)$

$\mu = np = 3(.5) = 1.5$
$\sigma^2 = np(1 - p) = 3(.5)(.5) = .75$
Final Presentation

- 10 minutes per person
- Any topic on statistical application
- PPT (or PDF) presentation
  - Memory stick
  - e-mail the file to me by Monday of the week
- Handout
  - Prepare 21 hard copies
  - e-mail the handout to me by Monday of the week
- Decide the topic and let me know by March 20.
  (10% of the final grade)
- Schedule
  - 3/27: 4 students
  - 4/10: 5 students
  - 4/17: 5 students
  - 4/24: 5 students
  - 5/1: 2 students