AMS595 Final project
Solve an ordinary differential equation with initial values

Try to solve the following equation numerically:

$$y'(t) = f(t, y), y(t_0) = y_0$$

Here we define $f(t, y) = sin(t) * cos(y)$,$t \in [0, 12], t_0 = 0, y_0 = y(t_0) = 0.1$

Requirements:

1. Try to use Euler method, Heun's method(Modified Euler method), and Midpoint method to implement it numerically. You can check wikipedia for definitions of these methods. The basic formulas are:
   Euler Method:
   $$y_{n+1} = y_n + h * f(t_n, y_n).$$
   Heun's Method:
   $$\hat{y}_{n+1} = y_n + h * f(t_n, y_n).$$
   $$y_{n+1} = y_n + \frac{h}{2}(f(t_n, y_n) + f(t_{n+1}, \hat{y}_{n+1})).$$
   Midpoint Method:
   $$y_{n+1} = Y_n + h * f(t_n + \frac{h}{2}, y_n + \frac{h}{2} f(t_n, y_n))).$$

2. You can use C/C++, Matlab, or Mathematica(other tools are not allowed) to wirte the program.

3. You have to write a two-page report with latex. In this report, plot your solutions, show the convergence(as n(number of steps) increases to a very large number,which means when the step length decreases, the solution will not change, you can use n=100,200,400,800, ...) of these methods, compare them(convergence speed), and try to give some conclusions. Also you can use mathematica to calculate the analytical solution for given equation, and compute errors.

4. Your project should be finished by yourself and have three parts: your code(c/c++, matlab, or mathematica), your latex source files for your report, and your report(pdf).

5. Make a **.tar** file and send it to **wli@ams.sunysb.edu** as an attachment.

6. Due date: 12pm(Noon, not midnight!). Dec. 15, 2011, Thurs. No late submission accepted.

Hints:
First divide the domain $[t_0, T]$ into n segments and then calculate solutions on those n+1 points($t_0, t_1, ..., t_n$,$t_n = T$).
Since $y_0 = y(t_0)$ is known, from the formula of euler method, we can calculate y_1 by $y_1 = y_0 + h * f(t_0, y_0)$, similarly, you can get $y_2, y_3, ..., y_n$. Now you have solutions based on n+1 points. You can plot (t,y) to see what the solutions look like. You can increase the number of n to check the convergence.

Pseudo code for Euler method:
a = t_0;
b = T;
h = (b-a)/n;
for i = 1:n
$t_i = a + i * h$;
$y_i = y_{i-1} + h * f(t_{i-1}, y_{i-1})$;
end