AMS 501 Homework #5

Due: Monday 11/28/2011

1. (15 points) Show that the equation
   \[ x^2 y'' + x^2 y' + y = 0 \]
   has no power series solution of the form \( y = \sum c_n x^n \).

2. (15 points) Find the Taylor series about 0 of the solution to the initial-value problem
   \[ y'' - 2xy' + 8y = 0; \quad y(0) = 4, \quad y'(0) = 0. \]
   Where might one expect the series to converge?

3. (20 points) Solve the initial value problem
   \[ y'' + xy' + (2x^2 + 1)y = 0; \quad y(0) = 1, \quad y'(0) = -1. \]
   Determine sufficiently many terms to compute \( y(1/2) \) accurately to four decimal places.

4. (30 points) Find two linearly independent Frobenius series solutions (for \( x > 0 \)) of following equations:
   (a) \( 2xy'' + 3y' - y = 0 \);
   (b) \( 2xy'' + (1 - 2x^2)y' - 4xy = 0 \).

5. (20 points) Note that \( x = 0 \) is an irregular singular point of the equation
   \[ x^2 y'' + (3x - 1)y' + y = 0. \]
   (a) Show that \( y = x^r \sum_{n=0}^{\infty} c_n x^n \) can satisfy this equation only if \( r = 0 \).
   (b) Substitute \( y = \sum_{n=0}^{\infty} c_n x^n \) to derive the solution. What is the radius of convergence of this series?