1. (20 points) Solve the differential equation

\[ x^2 y'' - xy' + (x^2 + 1)y = 0. \]

Find the first four nonzero terms in a Frobenius series solution, and then use the reduction of order technique to find the logarithmic term and the first three nonzero terms of a second linearly independent solution.

2. (20 points) Find two linearly independent solutions of Bessel’s equation,

\[ x^2 y'' + xy' + (x^2 - \frac{25}{4})y = 0. \]

3. (20 points) Express the general solution of each of the following differential equations in terms of Bessel functions.

(a) \( xy'' + 3y' + xy = 0 \)

(b) \( x^2 y'' - 5xy' + (8 + x)y = 0 \).

4. (40 points) Find the leading behaviors as \( x \to 0^+ \) of the following equations:

(a) \( y'' = \sqrt{x}y \);

(b) \( x^4 y'' - x^2 y' + \frac{1}{2}y = 0 \).