AMS 501 Sample Questions for Test 2

October 28, 2011

1. (25 points) Consider the initial value problem

\[ x' = Ax + f(t), \quad x(a) = x_a, \]

where

\[ A = \begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}, \quad f(t) = \begin{bmatrix} 0 \\ t^{-2} \end{bmatrix}, \quad x(1) = \begin{bmatrix} 3 \\ 7 \end{bmatrix}. \]

Solve it using the method of variation of parameters.

2. (25 points) Find the general solution of the following Riccati equation

\[ y' + y^2 = 1 + x^2, \]

given that \( y = x \) is a solution.

3. (30 points)

(a) Find the eigenvalues and eigenfunctions to the following boundary value problems

\[ y'' + \lambda y = 0, \quad y(0) = y'(\pi) = 0. \]

(b) Represent function \( f(x) = x \) as a series of eigenfunctions of the above problem.

4. (20 points) Classify all the singular points (finite and infinite) of the differential equation

\[ xy'' + (b - x)y' - ay = 0, \]

where \( a \) and \( b \) are constants.