Note: The exam is closed-book. However, you can have a single-sided, one-page, letter-size cheat sheet.

1. Answer true or false and give a **brief justification**. (No credit without justification.)
   
   (a) Whether an algorithm for a given problem is stable, backward stable, or unstable is independent of whether the problem is well-conditioned for a given input.
   
   (b) Provided row interchanges are allowed, the LU factorization always exists for square matrices.
   
   (c) If $A \in \mathbb{R}^{n \times n}$ is symmetric positive definite, then so is $BAB^{-1}$ for any nonsingular $B \in \mathbb{R}^{n \times n}$.

2. Given matrices $A, B \in \mathbb{R}^{m \times n}$, answer whether the following statements are true or false and give a **brief** argument. (You will not get points if you do not give any justification.)

   (a) $\|A\|_1 \geq \|A\|_2$
   
   (b) $\|A\|_2 = 1/\|A^{-1}\|_2$ (assuming $m = n$ and $A$ is nonsingular)
   
   (c) $\|A\|_2 = \|A\|_F$

3. A matrix $A$ is strictly column diagonally dominant if $|a_{ii}| > \sum_{j \neq i} |a_{ji}|$ for all $i$. Show that such a matrix is nonsingular.

4. Assume the following algorithms are implemented on a computer satisfying the two floating-point axioms. For each algorithm, state whether it is **backward stable**, **stable but not backward stable**, or **unstable**, and explain why.

   (a) (10 points) Data: $x \in \mathbb{R}$. Solution: $1 - x$, computed as $\text{fl}(1) \ominus \text{fl}(x)$.
   
   (b) (10 points) Data: $x \in \mathbb{R}$. Solution: $0.5x$, computed as $\text{fl}(x) \otimes 0.5$.

5. The following pseudo-code computes the Cholesky factorization $A = R^TR$, where $A$ is symmetric positive definite and $R$ is an upper triangular matrix.

   (a) Fill in the two blank lines in the algorithm to make it complete.

   ```plaintext
   Cholesky factorization
   $R = A$
   for $k = 1 : n$
     for $j = k + 1 : n$
       end
   end
   ```

   (b) What is the number of flops of this algorithm? Give only the leading term.