AMS 526 Homework 1, Fall 2008

Due: Tuesday 09/24 in class.

1. (15 points) Show that if a matrix $\boldsymbol{A}$ is both triangular and unitary, then it is diagonal.

2. (30 points) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are in $\mathbb{C}^m$, then the matrix $\boldsymbol{A} = \boldsymbol{I} + \boldsymbol{u}\boldsymbol{v}^*$ is called a *rank-one perturbation to the identity matrix*. Show that if $\boldsymbol{A}$ is nonsingular, then its inverse has the form $\boldsymbol{A}^{-1} = \boldsymbol{I} + \alpha\boldsymbol{u}\boldsymbol{v}^*$ for some scalar α, and give an expression for α. For what $\boldsymbol{u}$ and $\boldsymbol{v}$ is $\boldsymbol{A}$ singular? If it is singular, what is null($\boldsymbol{A}$)?

3. (20 points) Verify that $\|\boldsymbol{x}\boldsymbol{y}^*\|_F = \|\boldsymbol{x}\boldsymbol{y}^*\|_2 = \|\boldsymbol{x}\|_2\|\boldsymbol{y}\|_2$ for any $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{C}^m$.

4. (15 points) Two matrices $\boldsymbol{A}, \boldsymbol{B} \in \mathbb{C}^{m\times m}$ are *unitarily equivalent* if $\boldsymbol{A} = \boldsymbol{Q}\boldsymbol{B}\boldsymbol{Q}^*$ for some unitary $\boldsymbol{Q} \in \mathbb{C}^{m\times m}$. Is it true or false that $\boldsymbol{A}$ and $\boldsymbol{B}$ are unitarily equivalent if and only if they have the same singular values?

5. (20 points) We proved in class that every $\boldsymbol{A} \in \mathbb{C}^{m\times n}$ has an SVD $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^*$, where $\boldsymbol{U} \in \mathbb{C}^{m\times m}$ and $\boldsymbol{V} \in \mathbb{C}^{n\times n}$. It is true that if $\boldsymbol{A}$ is real, then it has a real SVD $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$ where $\boldsymbol{U} \in \mathbb{R}^{m\times m}$ and $\boldsymbol{V} \in \mathbb{R}^{n\times n}$? Prove or disprove.