AMS 526 Homework 5

Due: Tuesday 11/25 in class.

1. (20 points) Let $\rho(\boldsymbol{A})$ denote the *spectral radius* of $\boldsymbol{A}$, i.e., the largest absolute value of eigenvalue of $\boldsymbol{A}$. Show that $\rho(\boldsymbol{A}) \leq \|\boldsymbol{A}\|$ for any matrix norm induced by vector p-norm.

2. (20 points) Question 24.1(a,c,e,g) on page 188 of textbook.

3. (20 points) Question 26.1 on page 200 of textbook.

4. (40 points) In this exercise, you need to write a C code to call LAPACK routines `dgesvd` and `dsyev` to perform singular value decomposition and eigenvalue decomposition, and then use them to solve least squares problems $\boldsymbol{Ax} \approx \boldsymbol{b}$. Specifically, your code should use dgesvd to obtain the SVD of $\boldsymbol{A}$ and apply Algorithm 11.3 to solve $\boldsymbol{Ax} \approx \boldsymbol{b}$. In addition, use `dsyev` to obtain the eigenvalues and eigenvectors of $\boldsymbol{A}^T\boldsymbol{A}$, and then use them to solve the normal equation $\boldsymbol{A}^T\boldsymbol{Ax} = \boldsymbol{A}^T\boldsymbol{b}$. You can find the interface definitions for `dgesvd` and `dsyev` by searching at `http://netlib.org/cgi-bin/search.pl`. Also see the sample LAPACK code from lecture on 11/14 for examples of calling LAPACK in C.
   Use your C code to solve the least squares problem as in Question 4 of HW3. The problem arose from polynomial fitting of degree $n-1$,

   $$p_{n-1}(t) = x_1 + x_2 t + x_3 t^2 + \cdots + x_n t^{n-1},$$

   from m data points (t_i, y_i), $m > n$. Let $t_i = (i-1)/(m-1)$, $i = 1, \ldots m$, so that the data points are equally spaced on the interval $[0, 1]$. We will generate the corresponding values y_i by first choosing values for the x_j, say $x_j = 1$, $j = 1, \ldots, n$, and evaluating the resulting polynomial to obtain $y_i = p_{n-1}(t_i)$, $i = 1, \ldots, m$. The objective is to see whether we can recover the x_j that are used to generate y_i and compare the errors with those obtained in HW2. Choose $n = 3, 4, \ldots, 15$ and $m = 2n$, and plot the 2-norm errors using SVD and eigenvalue decomposition, along with the errors from Householder QR factorization from HW2. Start you implementation from the sample solution code for HW3. Submit your modified C code, the plots, and your conclusions of the comparative study.