

# AMS 526 Sample Questions for Test 1

October 9, 2008

1. Given matrices  $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{m \times n}$ , show whether the following statements are true or false:
  - (a)  $\|\mathbf{A}\|_1 \geq \|\mathbf{A}\|_2$
  - (b)  $\|\mathbf{A}\|_2 = 1/\|\mathbf{A}^+\|_2$  (assuming  $m \geq n$  and  $\mathbf{A}$  as full rank)
  - (c)  $\|\mathbf{A}\|_2 = \|\mathbf{A}\|_F$
2. Given a nonzero vector  $\mathbf{v} \in \mathbb{C}^m$ , let  $\mathbf{P} = \mathbf{I} + \alpha \mathbf{v} \mathbf{v}^*$ . For what nonzero scalar  $\alpha$  is  $\|\mathbf{P}\mathbf{x}\|_2 = \|\mathbf{x}\|_2$  for  $\mathbf{x} \in \mathbb{C}^m$ ? What is the geometric interpretation of  $\mathbf{P}$ ? Describe an efficient procedure to evaluate  $\mathbf{P}\mathbf{x}$ . What is the number of flops of the procedure?
3. Given a matrix  $\mathbf{A} \in \mathbb{C}^{m \times n}$  with full rank, its pseudoinverse is  $\mathbf{A}^+ = (\mathbf{A}^* \mathbf{A})^{-1} \mathbf{A}^*$  if  $m \geq n$  and is  $\mathbf{A}^+ = \mathbf{A}^* (\mathbf{A} \mathbf{A}^*)^{-1}$  if  $m \leq n$ . Express  $\mathbf{A}^+$  based on its SVD for both cases.
4. Given a linear least squares problem  $\mathbf{A}\mathbf{x} \approx \mathbf{b}$  where  $\mathbf{A}$  has more rows than columns and has full rank, name an efficient and numerically stable method for solving this problem. Explain why you favor this method over other alternatives.
5. Suppose the  $m \times n$  matrix  $\mathbf{A}$  has the form

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix},$$

where  $\mathbf{A}_1$  is a nonsingular matrix of dimension  $n \times n$  and  $\mathbf{A}_2$  is an arbitrary matrix of dimension  $(m - n) \times n$ . Prove that  $\|\mathbf{A}^+\|_2 \leq \|\mathbf{A}_1^{-1}\|_2$ .