AMS 526 Sample Questions for Test 2

November 5, 2008

1. Answer true or false with a brief justification. (No credit without justification.)

   (a) The absolute condition number of a problem (or function) f at an input x is always greater than its relative condition number at x.

   (b) If a matrix $\boldsymbol{A} \in \mathbb{C}^{m\times m}$ has a large relative condition number, we can make $\boldsymbol{A}$ better conditioned by scaling it with an appropriate positive scalar $\alpha \in \mathbb{R}$.

   (c) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are symmetric positive definite, then so is $\boldsymbol{A}+\boldsymbol{B}$.

   (d) If a matrix $\boldsymbol{A} \in \mathbb{C}^{m\times m}$ is Hermitian positive definite, then $\boldsymbol{A}$ must have an LU factorization (without pivoting) where the diagonal entries of $\boldsymbol{L}$ are all ones and those of $\boldsymbol{U}$ are all positive.

   (e) Whether a problem is well-conditioned depends on whether it is being solved using single-precision or double-precision floating-point arithmetic.

   (f) Classical Gram-Schmidt QR factorization is not backward stable but modified Gram-Schmidt QR factorization is backward stable.

2. Suppose $\boldsymbol{A} \in \mathbb{C}^{m\times m}$ is a banded matrix with lower bandwidth p and upper bandwidth q, i.e., $a_{ij} = 0$ for $i - j > p$ and $j - i > q$. (For example, a tridiagonal matrix has lower bandwidth 1 and upper bandwidth 1.) Let $\boldsymbol{PA} = \boldsymbol{LU}$ be the LU factorization with partial pivoting of $\boldsymbol{A}$. What is the upper bandwidth of $\boldsymbol{U}$? If there is no pivoting, what is the lower bandwidth of $\boldsymbol{L}$ and upper bandwidth of $\boldsymbol{U}$?

3. Let $\boldsymbol{L} \in \mathbb{C}^{m\times m}$ be a unit lower triangular matrix with off-diagonal nonzeros only in the second column, i.e.

$$\boldsymbol{L} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & l_{32} & 1 & & \\ & \vdots & & \ddots & \\ & l_{m2} & & & 1 \end{bmatrix}$$

   (a) Compute the condition number of $\boldsymbol{L}$ in ∞-norm.

   (b) Compute the condition number of $\boldsymbol{L}$ in 1-norm.

   (c) What are the eigenvalues of $\boldsymbol{L}$?

4. Prove or give a counterexample: If $\boldsymbol{A}$ is a symmetric positive definite matrix, then $\boldsymbol{A}^{-1}$ is also symmetric positive definite.