AMS 526 Homework 1

Due: Wednesday 09/12 in class.

1. (10 points) (Exercise 1.1.20 on p. 8 of textbook) Consider matrices A, X, and B, partitioned as indicated.
$$A = \left[\begin{array}{r|rr} 1 & 3 & 2 \\ 2 & 1 & 1 \\ \hline -1 & 0 & 1 \end{array}\right] \quad X = \left[\begin{array}{r|rr} 1 & 0 & 1 \\ \hline 2 & 1 & 1 \\ -1 & 2 & 0 \end{array}\right] \quad B = \left[\begin{array}{r|rr} 5 & 7 & 4 \\ 3 & 3 & 3 \\ \hline -2 & 2 & -1 \end{array}\right]$$
Thus, for example, $A_{12} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ and $A_{21} = \begin{bmatrix} -1 \end{bmatrix}$. Show that $AX = B$ and $A_{i1}X_{1j} + A_{i2}X_{2j} = B_{ij}$ for $i,\ j = 1,\ 2$.

2. (15 points) If u and v are in $\mathbb{R}^n$, then the matrix $A = I + uv^T$ is called a *rank-one perturbation to the identity matrix*. Show that if $\boldsymbol{A}$ is nonsingular, then its inverse has the form $A^{-1} = I + \alpha uv^T$ for some scalar α, and give an expression for α. For what u and v is A singular? If it is singular, what is null(A)?

3. (15 points) Show that if R is a nonsingular $n \times n$ upper-triangular matrix, then R^{-1} is also upper-triangular. Analogously, if L is a nonsingular $n \times n$ lower-triangular matrix, then L^{-1} is also lower-triangular.

4. (15 points) (Exercise 1.4.60 on p. 49 of textbook) Carefully prove by induction that the Cholesky decomposition is unique: Suppose $A = R^TR = S^TS$, where R and S are both upper-triangular matrices with positive main-diagonal entries. Partition A, R and S conformably and prove that the parts of S must equal the corresponding parts of R.

5. (15 points) (Exercise 1.5.9 on p. 59 of textbook) Let R be an $n \times n$ upper-triangular matrix with semiband width s. Show that the system $Rx = y$ can be solved by back substitution in about $2ns$ flops. An analogous result holds for lower-triangular systems.

6. (0 points) Programming assignment preparation: This part of the exercise is for you to get familiar with the computing environment. Unless you are proficient in setting up your own computer, you are urged to use the Linux (not Windows) computers at the Mathlab SINC Site in Math Tower S-235 to do all your computing assignments. Before you can log-on to the computers at the SINC site, you may need to go to the Mathlab SINC site in person to activate your account on any of the Linux system by following the on-screen instructions. After you log in, make sure you can run `emacs` (for editing), `gv` (for viewing eps files), and `ddd` (for debugging).

   If you have a personal computer, after activating your Mathlab account you can remotely log onto the computer "`compute.mathlab.sunysb.edu`" from your PC. You need an ssh client and XWindows server on your local computer (available on Linux, Mac OS X, and Cygwin on Microsoft Windows). You can remotely log onto the computer using command "`ssh -X -Y` *username*`@compute.mathlab.sunysb.edu`", where `username` should be changed to your own username, and -X and -Y options would ensure X Windows applications can be displayed on your local computer. After you remotely log onto the computer, try to run `gv`, `ddd`, and `emacs` and make sure the windows are properly displayed on your local computer. Practice using `scp` to copy files between you local and remote computers. Also, practice using the editor (emacs or vi) and the C compiler (gcc).

7. (30 points) Programming assignment. This part of the exercise is for you to implement a C program that performs ABx in two ways: $(AB)x$ and $A(Bx)$. Download the template code from http://www.ams.sunysb.edu/~jiao/teaching/ams526_fall12/HW/hw1_template.tgz, copy it to the Mathlab computer or your own Linux/Mac system, and run command "`tar zxvf hw1_template.tgz`". Implement the computational kernel of the program for matrix-vector and matrix-matrix multiplication by following the instruction in the template. Submit your completed code to the TA by email). Also submit a report for an analysis of the numbers of floating point operations required by the two approaches. Do your analysis correlates well with their actual performance?