AMS 526 Homework 5, Fall 2012

Due on Monday 11/19 in class

1. (10 points) Exercise 4.3.11 on p.278
2. (10 points) Exercise 4.3.14 on p.279
3. (10 points) Exercise 5.2.2 on p.305
4. (10 points) Exercise 5.2.8 on p.307
5. (10 points) Exercise 5.3.13 on p.319
6. (10 points) Exercise 5.4.27 on p.343
7. (10 points) Exercise 5.4.40 on p.345
8. (30 points) Write a C code to call LAPACK routines `dgesvd` and `dsyev` to perform singular value decomposition and eigenvalue decomposition, and then use them to solve least squares problems $Ax \approx b$. Specifically, your code should use `dgesvd` to obtain the SVD of A and apply Algorithm 1 (Algorithm 11.3 from Trefthen-Bau book) given below to solve $Ax \approx b$. In addition, use `dsyev` to obtain the eigenvalues and eigenvectors of $A^T A$, and then use them to solve the normal equation $A^T A x = A^T b$. You can find the interface definitions for `dgesvd` and `dsyev` by searching at `http://netlib.org/cgi-bin/search.pl`. Use your C code to solve the least squares problem arising from polynomial fitting of degree $n-1$,

   $$p_{n-1}(t) = x_1 + x_2 t + x_3 t^2 + \cdots + x_n t^{n-1},$$

   from m data points (t_i, y_i), $m > n$. Let $t_i = (i-1)/(m-1)$, $i = 1, \ldots m$, so that the data points are equally spaced on the interval $[0, 1]$. We will generate the corresponding values y_i by first choosing values for the x_j, say $x_j = 1$, $j = 1, \ldots, n$, and evaluating the resulting polynomial to obtain $y_i = p_{n-1}(t_i)$, $i = 1, \ldots, m$. The objective is to see whether we can recover the x_j that are used to generate y_i. Choose $n = 3, 4, \ldots, 15$ and $m = 2n$, and plot the 2-norm errors using SVD and eigenvalue decomposition. Submit your modified C code, the plots, and your conclusions of the comparative study.

Algorithm 1 Least Squares via SVD.

(a) Compute the reduced SVD $A = \hat{U}\hat{\Sigma}V^*$.

(b) Compute the vector $\hat{U}^* b$.

(c) Solve the diagonal system $\hat{\Sigma} w = \hat{U}^* b$ for w.

(d) Set $x = Vx$.
