AMS 526 Homework 6, Fall 2012

Due: Wednesday 12/5 in class

1. (10 points) Question 5.5.1 on page 351 of textbook.
2. (10 points) Question 5.8.9 on page 390 of textbook.
3. (10 points) Question 6.3.20(a) on page 435 of textbook.
4. (10 points) Question 6.4.21 on page 446 of textbook.
5. (10 points) Question 7.1.7 on page 473 of textbook.
6. (10 points) Question 7.1.14 on page 476 of textbook.
7. (10 points) Let $\boldsymbol{A} \in \mathbb{C}^{m \times m}$ be a Hermitian matrix and $\boldsymbol{q} \in \mathbb{C}^m$ be a vector with $\|\boldsymbol{q}\|_2 = 1$.
   (a) Show that there exists a unitary matrix $\boldsymbol{Q} \in \mathbb{C}^{m \times m}$ whose **first** column is $\boldsymbol{q}$, such that $\boldsymbol{Q}^* \boldsymbol{A} \boldsymbol{Q}$ is a tridiagonal matrix.
   (b) Prove or disprove: There exists a unitary matrix $\tilde{\boldsymbol{Q}} \in \mathbb{C}^{m \times m}$ whose **last** column is $\boldsymbol{q}$, such that $\tilde{\boldsymbol{Q}}^* \boldsymbol{A} \tilde{\boldsymbol{Q}}$ is a tridiagonal matrix.
8. (10 points) The preliminary reduction to tridiagonal form would be of little use if the steps of the QR algorithm did not preserve this structure. Fortunately, they do.
   (a) In the QR factorization $A = QR$ of a symmetric tridiagonal matrix A, which entries of R are in general nonzero? Which entries of Q? (In practice we do not form Q explicitly.)
   (b) Show that the tridiagonal structure is recovered when the product is RQ is formed.
9. (10 points) Show that if the largest off-diagonal entry is annihilated at each step of the Jacobi algorithm, then the sum of the squares of the off-diagonal entries decreases by at least the factor $1 - 2/(m^2 - m)$ at each step?
10. (10 points) Question 8.8.13 on page 611 of textbook.