AMS 526 Sample Questions for Test 1

September 20, 2012

Note: The exam is closed-book. However, you can have a single-sided, one-page, letter-size cheat sheet.

1. Answer true or false and give a **brief justification**. (No credit without justification.)

   (a) Whether an algorithm for a given problem is stable, backward stable, or unstable is independent of whether the problem is well-conditioned for a given input.

   (b) Provided row interchanges are allowed, the LU factorization always exists for square matrices.

   (c) If $A \in \mathbb{R}^{m\times m}$ is symmetric positive definite, then so is BAB^{-1} for any nonsingular $B \in \mathbb{R}^{n\times n}$.

2. Given matrices $A, B \in \mathbb{R}^{m\times n}$, answer whether the following statements are true or false and give a **brief** argument. (You will not get points if you do not give any justification.)

   (a) $\|A\|_1 \geq \|A\|_2$

   (b) $\|A\|_2 = 1/\|A^{-1}\|_2$ (assuming $m = n$ and A is nonsingular)

   (c) $\|A\|_2 = \|A\|_F$

3. A matrix A is called strictly column diagonally dominant if $|a_{ii}| > \sum_{j\neq i} |a_{ji}|$ for all i. Show that such a matrix is nonsingular. (Hint: A has LU factorization.)

4. Assume the following algorithms are implemented on a computer satisfying the two floating-point axioms. For each algorithm, state whether it is *backward stable*, *stable but not backward stable*, or *unstable*, and explain why.

   (a) (10 points) Data: $x \in \mathbb{R}$. Solution: $1 - x$, computed as $\mathrm{fl}(1) \ominus \mathrm{fl}(x)$.

   (b) (10 points) Data: $x \in \mathbb{R}$. Solution: $0.5x$, computed as $\mathrm{fl}(x_1) \otimes 0.5$.

5. The following pseudo-code computes the Cholesky factorization $A = R^T R$, where A is symmetric positive definite and R is an upper triangular matrix.

   (a) Fill in the two blank lines in the algorithm to make it complete.

   ```
   Cholesky factorization
     R = A;
     for k = 1 to n
           for j = k + 1 to n
                 ______________________________
           end
           ________________________________
      end
   ```

   (b) What is the number of flops of this algorithm? Give only the leading term.