

AMS 526 Sample Questions for Test 1

September 26, 2018

Note: The exam is closed-book. However, you can have a single-sided, one-page, letter-size cheat sheet.

1. Answer true or false and give a **brief justification**. (No credit without justification.)
 - (a) Whether an algorithm for a given problem is stable, backward stable, or unstable is independent of whether the problem is well-conditioned for a given input.
 - (b) Provided row interchanges are allowed, the LU factorization always exists for square matrices.
 - (c) If $A \in \mathbb{R}^{n \times n}$ is symmetric positive definite, then so is BAB^{-1} for any nonsingular $B \in \mathbb{R}^{n \times n}$.
2. Given matrices $A, B \in \mathbb{R}^{m \times n}$, answer whether the following statements are true or false and give a **brief** argument. (You will not get points if you do not give any justification.)
 - (a) $\|A\|_1 \geq \|A\|_2$
 - (b) $\|A\|_2 = 1/\|A^{-1}\|_2$ (assuming $m = n$ and A is nonsingular)
 - (c) $\|A\|_2 = \|A\|_F$
3. A matrix A is strictly column diagonally dominant if $|a_{ii}| > \sum_{j \neq i} |a_{ji}|$ for all i . Show that such a matrix is nonsingular.
4. Assume the following algorithms are implemented on a computer satisfying the two floating-point axioms. For each algorithm, state whether it is *backward stable*, *stable but not backward stable*, or *unstable*, and explain why.
 - (a) (10 points) Data: $x \in \mathbb{R}$. Solution: $1 - x$, computed as $\text{fl}(1) \ominus \text{fl}(x)$.
 - (b) (10 points) Data: $x \in \mathbb{R}$. Solution: $0.5x$, computed as $\text{fl}(x_1) \otimes 0.5$.
5. The following pseudo-code computes the Cholesky factorization $A = R^T R$, where A is symmetric positive definite and R is an upper triangular matrix.
 - (a) Fill in the two blank lines in the algorithm to make it complete.

```
Cholesky factorization
R = A
for k = 1 : n
    for j = k + 1 : n
        _____
    end
    _____
end
```
 - (b) What is the number of flops of this algorithm? Give only the leading term.