AMS 526 Sample Questions for Test 2

Note: The exam is closed-book. However, you can have a single-sided, one-page, letter-size cheat sheet.

1. Answer true or false and give a **brief justification**. (No credit without justification.)

   (a) When solving the least squares problem $Ax \approx b$ using a backward stable algorithm, if the perturbations in A is $O(\epsilon)$, then the error in the output is $O(\kappa(A)\epsilon)$.

   (b) If $A \in \mathbb{R}^{n \times n}$ is symmetric positive definite, then BAB^T has the same eigenvalues as A for any nonsingular $B \in \mathbb{R}^{n \times n}$.

   (c) Let λ be an eigenvalue of an upper triangular matrix $A \in \mathbb{C}^{n \times n}$. The algebraic multiplicity and geometric multiplicity of λ as the eigenvalue of A must be equal.

2. Let $A \in \mathbb{R}^{n \times n}$ be a matrix with rank $n - 1$. Suppose $AP = QR$ is the QR factorization of A with column pivoting, where P is a permutation matrix such that the diagonal entry r_{kk} of R is maximized at the kth step. Ignoring the effect of rounding errors, show that $r_{nn} = 0$ and $r_{n-1,n-1} \neq 0$.

3. Given a linear least squares problem $Ax \approx b$ where A has more rows than columns and has full rank, name an efficient and numerically stable method for solving this problem. Explain why you favor this method over other alternatives.

4. Suppose that all the row sums of a matrix $A \in \mathbb{C}^{n \times n}$ have the same value, say α. Show that α is an eigenvalue of A. What is the corresponding eigenvector?

5. Let $A \in \mathbb{C}^{n \times n}$, and $A = B + iC$, where $B, C \in \mathbb{R}^{n \times n}$.

   (a) Show that A is Hermitian if and only $M = \begin{bmatrix} B & -C \\ C & B \end{bmatrix}$ is symmetric.

   (b) Suppose A is Hermitian. Express the eigenvalues and eigenvectors of M in terms of those of A.