AMS527: Numerical Analysis II

Review for Final Exam

Xiangmin Jiao

SUNY Stony Brook

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Announcements

- Final exam will be on Wed 5/13 at 8:00–10:30am in P-128 Physics Building
- The exam will be comprehensive
  - about 50% on materials after Test 2
  - about 30% on materials after Test 1 and before Test 2
  - about 20% on materials before Test 1

Materials Covered Since Test 2

- Two-point boundary value problems (BVP)
  - Concepts: existence and uniqueness (for linear BVP), conditioning and stability, types of boundary conditions
  - Numerical methods for solving two-point BVPs: shooting method, **finite difference**, collocation, Galerkin
  - (Not required: eigenvalue problems for ODEs)
- Overview of partial differential equations
  - Classification of PDEs, example equations
  - Time-dependent problems,
    - Concepts: stiffness, Lax Equivalence theorem, stability, CFL condition
    - Numerical methods, **semidiscrete methods** (**method of lines**), Crank-Nicolson method,
  - Finite-difference methods for time-independent PDEs

Materials Covered Since Test 2

- Finite element methods for Poisson equation
  - Concepts: strong form vs. weak form, **Galerkin's method vs. Rayleigh-Ritz method**, shape functions, general procedure for elemental computation, accuracy and convergence
- Sparse linear systems
  - Concepts: sparsity and their origins, direct methods (fills), iterative methods (mostly relaxation methods),
  - **Multigrid methods** (motivation, residual equation, restriction, injection, efficiency)
- Fourier transform
  - Concept: trigonometric interpolation, discrete Fourier transform (DFT), inverse discrete Fourier transform, analogy with polynomial interpolation, limitations, applications to convolution
  - Algorithm: **fast Fourier transform** (FFT), inverse fast Fourier transform

Sample Questions

- Some sample questions from text book
  - BVP: 10.2(a), 10.4(b)
  - PDEs: 11.2, 11.3,
  - FFT: 12.8, 12.9
- More sample questions to be posted online

Materials Covered Before Test 1

- Approximations in scientific computations
  - Concepts: absolute error, relative error, computational error, propagated data error, truncation error, rounding error, forward error, backward error, condition number, stability, cancellation
- Solutions of nonlinear equations
  - Concepts: multiplicity, sensitivity, convergence rate
  - Basic algorithms: Interval bisection method, **fixed-point iteration**
  - **Newton's method**, secant method, Broyden's method, and other Newton-like method

Materials Covered Before Test 1

- Numerical optimization
  - Concepts: unconstrained optimization, constrained optimization (linear vs. nonlinear programming), global vs. local minimum, coercive, convex, first- and second-order optimality condition, unimodality
  - Algorithms for unconstrained optimization: golden section search, **Newton's method**, Quasi-Newton methods (basic ideas), **steepest descent**, conjugate gradient (basic ideas)
  - Lagrange multiplier for constrained optimization, especially equality-constrained optimization
- Interpolation
  - Concepts: interpolation vs. approximation, basis functions, orthogonal polynomials, convergence, **Taylor polynomial**
  - **Polynomial interpolation** using monomial, Lagrange, Newton basis functions, comparisons, divided difference
  - Orthogonal polynomial interpolation, properties

Materials Covered Before Test 2

- Numerical Integration
  - Concepts: quadrature rules; degree of quadrature rules; stability of quadrature rules; connections with polynomial interpolation; Newton-Cotes rules; Gaussian quadrature rules; change of interval; composite quadrature rule; multiple integrals
  - Basic algorithms/schemes: **method of undetermined coefficients**; midpoint rule, trapezoid rule; Simpson's rule; **Gaussian quadrature rules**
  - Application to integral equations
- Numerical Differentiation
  - Concepts: **Finite difference approximation**; connections with polynomial interpolation
  - Basic algorithms/schemes: forward difference, backward difference, centered difference
- Other key concepts: Integral equations and Richardson extrapolation

Materials Covered Before Test 2

- Numerical Methods for Initial Value Problems
  - Concepts: existence, uniqueness, and stability of solutions of ODEs; order of ODEs; global error vs. local error; growth factor for analysis of stability; stiffness; unconditionally stable; predictor-corrector methods
  - Basic algorithms/schemes: Euler's method; backward Euler; **trapezoid method**; Heun's method; fourth-order **Runge-Kutta method**
  - **Understand how first- and second-order methods are derived**
  - Other methods: Taylor series methods; multistep methods; multivalue methods