AMS 527, Spring 2012, Homework 7

100 points. Due: Wednesday 05/02

Electronic submission of homework assignments is strongly encouraged for the computer problems. For the written part, you are encouraged (but not required) to typeset using \LaTeX or \textsc{LyX} (an easy-to-use front-end of \LaTeX). For electronic submission, homework is due at 11:59pm on the due date. For paper submission, homework is due in class on the due date.


2. (15 points) Exercise 11.4 on page 490 of Heath book.

3. (1 points) Given the Poisson’s equation in 2-D

\[ u_{xx} + u_{yy} = f(x, y), \quad 0 < x, y < 1, \]

with boundary conditions

\[ u(x, y) = 0, \quad x = 0, 1. \]
\[ u(x, y) = 1, \quad y = 0, 1. \]

Approximate its solution by \( u(x, y) \approx \sum_{i=1}^{n} s_i \phi_i(x, y) \), where \( \phi_i(t) \) is the basis function.

(a) Use Galerkin’s method to derive the weak form

\[ -\sum_{j=1}^{n} \int_{S} \phi_i(x, y) \frac{\partial \phi_j(x, y)}{\partial x} \, dx \, dy + \int_{S} \frac{\partial \phi_i(x, y)}{\partial y} \frac{\partial \phi_j(x, y)}{\partial y} \, dx \, dy = \int_{0}^{1} \int_{0}^{1} f(x, y) \phi_i(x, y) \, dx \, dy \quad (1) \]

What conditions does \( \phi_j(x, y) \) need to satisfy for the weak form to be valid?

(b) Show how to use Rayleigh-Ritz method (i.e., minimum principle) to derive the weak form (1).


5. (15 points) FFT Exercise 12.10 on page 508 of Heath book.