1. Let $X$ and $Y$ be two continuous r.v.’s with joint density function

$$f(x, y) = \begin{cases} \frac{3}{2}(x^2 + y^2) & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal distribution $f_X(x)$.

(b) Find $P(X \leq \frac{1}{2})$.

(c) Find the probability $P(X \leq \frac{1}{2}, Y \leq \frac{1}{2})$.

(d) Are $X$ and $Y$ independent? Justify your answer.

2. Stores A, B, and C have 50, 75, and 100 employees, and, respectively, 50, 60, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store C?

3. The number of storms in the rainy season is Poisson distributed but with a parameter value that is uniformly distributed over $(0, 5)$. That is, the parameter $\Lambda \sim U(0, 5)$. Given that $\Lambda = \lambda$, the number of storms is Poisson with mean $\lambda$. Find the probability there are at least three storms this season.

4. Suppose that customers arrive at a bank according to a $PP(\lambda)$ with $\lambda = 12$ per hour. Compute the following:

(a) The mean and variance of the customers who enter the bank during 5 hours.

(b) Probability that more than 5 customers enter the bank during an hour.

(c) Probability that exactly 1 arrival between 9:00am and 11:00am and exactly 2 arrivals between 10:00am and 12:00 noon.

5. A system consists of $n$ components in parallel. The lifetimes of the components are i.i.d. $exp(\lambda)$ random variables. The system functions as long as at least one of the $n$ components is functioning. Let $T$ be the lifetime of the system. Compute $E[T]$.

6. Let $\{N(t), t \geq 0\}$ be a $PP(\lambda)$. Suppose a Bernoulli switching mechanism independently marks the events as either type 1 or type 2 with probability $p$ and $1 - p$, respectively. Let $N_i(t)$ be the number of type $i$ events during $(0, t]$. Let $T_i$ be the time until the first event in the $N_i(t)$ process. Compute the joint distribution of $(T_1, T_2)$.

7. Let $\{N(t), t \geq 0\}$ be a $PP(\lambda)$. Compute

$$P[N(t) = k | N(t + s) = k + m] \quad \text{for } t \geq 0, s \geq 0, k \geq 0, m \geq 0.$$