

AMS507 Introduction to Probability. First Midterm.

October 13, 2009

Your name:

Your ID number:

You shall receive 10 points for each of the following 10 question. The total will be divided by 10. The maximum score is 10.

1. How many hands of 13 cards dealt from a normal shuffled pack of 52 cards contain exactly two kings and one ace.

$$\binom{4}{2} \binom{4}{1} \binom{44}{10}$$

2. If there are no restrictions on where the digits and letters are places. How many 8-place license plates consisting of 5 letters and 3 digits are possible if no repetitions of letters or digits are allowed?

$$\binom{26}{5} \binom{10}{3} \cdot 8!$$

3. A fair coin is thrown repeatedly. What is the probability that on the n th throw: (a) a head appears for the first time? (b) the number of heads equal the number of tails?

(a) $\left(\frac{1}{2}\right)^n$

(b) if n is odd, the prob is 0.

if n is even, the prob is $\binom{n}{\frac{n}{2}} \left(\frac{1}{2}\right)^n$

4. Six balls are to be randomly chosen from an urn containing 8 red, 10 green, and 12 blue balls. Given that no red balls are chosen, what is the probability that there are exactly 2 green balls among the 6 chosen?

$$\frac{\binom{10}{2} \binom{12}{4}}{\binom{22}{6}}$$

5. Balls are randomly removed from an urn initially containing 20 red and 10 blue balls. What is the probability that all of the red balls are removed before all of the blue ones have been removed?

$\{ \text{all red balls removed before all blue ones} \} \equiv \{ \text{the last one removed is blue} \}$.

$$P(\{ \text{the last removed one is blue} \}) = \frac{1}{3}$$

6. An airline company knows that 5 percent of the people buying tickets on a certain flight will not show up. The company has just sold 52 tickets for a flight that can hold only 50 passengers. What is the probability that there will be a seat available for every passenger who shows up?

Let $X = \#$ passengers who show up

$$X \sim \text{Bin}(52, 0.95)$$

$$P(X \leq 50) = 1 - P(X=51) - P(X=52) = 1 - \binom{52}{51} 0.95^{51} \cdot 0.05 - \binom{52}{52} 0.95^{52}$$

7. A bowl contains 20 cherries, exactly 15 of which have their stones removed. A greedy pig eats 2 whole cherries, picked at random, without remarking on the presence or absence of stones. Subsequently, a cherry is picked randomly from the remaining 18. Given that this cherry contains a stone, what is the probability that the pig consumed at least one stone?

Let $A_i = \{ \text{the two cherries in the pig contain } i \text{ stones} \}, i=0,1,2$.

$B = \{ \text{the randomly picked cherry contains a stone} \}$

Clearly $P(B) = \frac{1}{4}$

$$P(A_0|B) = \frac{P(B|A_0)P(A_0)}{P(B)} = \frac{\frac{5}{18} \cdot \frac{\binom{15}{2}\binom{5}{0}}{\binom{20}{2}}}{\frac{1}{4}} \Rightarrow P(\text{at least one stone}) = 1 - P(A_0|B) = 1 - \frac{\frac{5}{18} \cdot \frac{\binom{15}{2}\binom{5}{0}}{\binom{20}{2}}}{0.25}$$

8. We toss n coins, and each toss shows heads with probability p , independently of each of the others. Each coin which shows heads is tossed again. What is the distribution function of the number of heads resulting from the second round of tosses?

$X = \#$ of heads resulting from the second round

$$P(X=k) = \binom{n}{k} p^{2k} (1-p)^{n-k}$$

9. If $P(A)$ is either 0 or 1, show that event A is independent of all events

B. if $P(A) = 0$, then $P(A \cap B) \leq P(A) = 0 \Rightarrow P(A \cap B) = 0$
 $\Rightarrow P(A \cap B) = P(A)P(B) \Rightarrow A \perp B.$

if $P(A) = 1$, then $1 = P(A) \leq P(A \cup B) \Rightarrow P(A \cup B) = 1$
 $\Rightarrow P(A) + P(B) - P(A \cap B) = 1 \Rightarrow P(A \cap B) = P(B) = P(A)P(B)$
 $\Rightarrow A \perp B$

10. Suppose that every lottery ticket has a probability $p = 10^{-6}$ of winning a big jackpot, independently of all other lottery tickets. Suppose you buy 500,000 tickets. Estimate the probability that exactly 5 of them win the big jackpot.

$X = \#$ of winning tickets

X is binomial with parameters $p = 10^{-6}$ $n = 500,000$.

Therefore X is approximately Poisson with parameter

$$\lambda = np = 0.5$$

$$P(X=5) = e^{-0.5} \frac{(0.5)^5}{5!}$$