

AMS 553: Homework 1

1. (L&K 4.2) Let X be a continuous random variable with pdf

$$f(x) = x^2 + \frac{2}{3}x + \frac{1}{3} \quad \text{for } 0 \leq x \leq c$$

- (a) Find the value of c ; (b) Plot the pdf $f(x)$; (c) Compute and plot the cdf $F(x)$; (d) Compute $P(\frac{1}{3} \leq X \leq \frac{2}{3})$, $E[X]$, and $Var(X)$.

2. (L&K 4.3) Suppose that X and Y are jointly discrete random variables with

$$p(x, y) = \begin{cases} \frac{2}{n(n+1)} & \text{for } x = 1, 2, \dots, n \text{ and } y = 1, 2, \dots, x, \\ 0 & \text{otherwise.} \end{cases}$$

Compute the pmfs $p_X(x)$ and $p_Y(y)$ and determine whether X and Y are independent.

3. (L&K 4.6) Suppose X and Y are jointly continuous random variables with density function

$$f(x, y) = \begin{cases} 32x^3y^7 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Compute $f_X(x)$ and $f_Y(y)$ and determine whether X and Y are independent.

4. (L&K 4.9) Suppose X is a discrete random variable with $p_X(x) = 0.25$ for $x = -2, -1, 1, 2$. Let Y also be a discrete random variable such that $Y = X^2$. Show that $Cov(X, Y) = 0$. Therefore, uncorrelated random variables are not necessarily independent.

5. (L&K 4.23) Suppose that 7.3, 6.1, 3.8, 8.4, 6.9, 7.1, 5.3, 8.2, 4.9, 5.8 are 10 observations taken from a distribution with unknown mean μ . Compute $\bar{X}(10)$, $S^2(10)$, and an approximate 95 percent confidence interval for μ .

6. (L&K 4.24) In problem 5, test the null hypothesis $H_0 : \mu = 6$ at significance level $\alpha = 0.05$.

7. (L&K 4.27) A geometric distribution with parameter $\rho \in (0, 1)$ has probability mass function

$$p(x) = \rho(1 - \rho)^x \quad \text{for } x = 0, 1, 2, \dots$$

Show that this distribution has the memoryless property.

8. Consider a single queue with the following interarrival and service times (in minutes) for the first ten customers:

interarrival: 3 3 2 4 6 2 4 8 4 5 5
 service times: 4.5 4 2 2 4 3 4 6 4 3

Let $N(t)$ be the number of customers in system up to time t . Assume that the system starts empty, i.e., $N(0) = 0$ and the “zeroth” arrival happens at time 0. Simulate the process $\{N(t)\}$ under the following cases:

- (i) single-server, FCFS queue discipline;
- (ii) two-server, FCFS queue discipline;
- (iii) single-server, shortest processing time (SPT) queue, where the next customer served is the one in queue with the shortest service time;

For each case, *draw* the sample path for two terminating conditions: (a) the completion of service for 10 customers; (b) 30 minutes total simulation time, and *compute* the following performance measures:

- (i) average time that a customer waits in queue;
- (ii) time-average number of customers in the queue;
- (iii) fraction of customers that spent more than 4.5 minutes in the system;
- (iv) fraction of time that the system is busy (utilization);
- (v) fraction of time that there is more than one customer in queue.