

AMS 553: Homework 2

1. (L&K 7.1) Consider the LCG $Z_i = (5Z_{i-1} + 3)(\text{mod}16)$ and $Z_0 = 7$. Find Z_{500} , using only pencil and paper.
2. (L&K 7.2) For the following multiplicative LCGs, compute Z_i for enough values of $i \geq 1$ to cover an entire cycle:
 - (a) $Z_i = (11Z_{i-1})(\text{mod} 16)$, $Z_0 = 1$
 - (b) $Z_i = (11Z_{i-1})(\text{mod} 16)$, $Z_0 = 2$
 - (c) $Z_i = (2Z_{i-1})(\text{mod} 13)$, $Z_0 = 1$
 - (d) $Z_i = (3Z_{i-1})(\text{mod} 13)$, $Z_0 = 1$

Note that (a) and (b) have m of the form 2^b ; (c) is a PMMLCG, for which $a = 2$ is a primitive element modulo $m = 13$.

3. (L&K 7.3) Without actually computing any Z_i 's, determine which of the following mixed LCGs have full period:
 - (a) $Z_i = (13Z_{i-1} + 13)(\text{mod} 16)$
 - (b) $Z_i = (12Z_{i-1} + 13)(\text{mod} 16)$
 - (c) $Z_i = (13Z_{i-1} + 12)(\text{mod} 16)$
 - (d) $Z_i = (Z_{i-1} + 12)(\text{mod} 13)$

For part (d), also plot the pairs $(U_1, U_2), (U_2, U_3), \dots$. Are there any obvious undesirable structure?

4. Without generating any Z_i 's, determine which of the following multiplicative LCGs is an PMMLCG?
 - (a) $Z_i = (2Z_{i-1})(\text{mod} 19)$
 - (b) $Z_i = (4Z_{i-1})(\text{mod} 15)$
 - (c) $Z_i = (97Z_{i-1})(\text{mod} 9021)$