AMS 553: Homework 4

1. Consider an inventory system with the following daily demand:

n :	1	2	3	4	5	6	7	8	9	10
D_n :	13	27	33	5	43	17	52	33	15	19

   An (s, S) ordering policy is used to manage the inventory system. At the beginning of each day, assume that the demand is subtracted first, then an order decision is made, and the order (if any) is filled in the next period (i.e., the order lead time is one day). Let I_n be the inventory level on day n and the initial inventory level $I_0 = 100$. Assume full backlog of demand, i.e., all demand cannot be satisfied immediately is backlogged and eventually filled. Simulate (*by hand*) the system by using the ordering policy $(50, 100)$ for 10 days. Plot the sample path for the inventory level and calculate the following performance measures:

   - the average inventory level.
   - average positive and negative inventory level.
   - the proportion of demand met from on-hand inventory (not backlogged).

2. (L&K 6.2) A random variable X is said to have Weibull distribution with parameters α and β if its density function is given by

$$f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha} \quad x > 0$$

   and its cumulative distribution function is given by

$$F(x) = 1 - e^{-(x/\beta)^\alpha} \quad x > 0.$$

   Show that $X \sim Weibull(\alpha, \beta)$ if and only if $X^\alpha \sim exp(\beta^\alpha)$.

3. (L&K 6.9) For a geometric distribution with parameter p, explain why the MLE $\hat{p} = 1/[\bar{X}(n) + 1]$ is intuitive.

4. (L &K 6.10) For each of the following distributions, derive formulas for the MLEs of the indicated parameters. Assume that we have iid data $X_1, X_2, \ldots X_n$ from the distribution in question

   (d) $N(\mu, \sigma^2)$, joint MLEs for μ and σ.

   (e) $LN(\mu, \sigma^2)$, joint MLES for μ and σ.

   (g) $DU(i, j)$, joint MLEs for i and j.

   (h) $bin(t, p)$, MLE for p assuming that t is known.

   (j) $U(\theta - 0.5, \theta + 0.5)$, MLE for θ.

5. (L &K 6.11) For a Poisson distribution with parameter λ, derive an approximate $100(1-\alpha)\%$ confidence interval for λ given the data $X_1, X_2, \ldots, X_n$. Use the asymptotic normality of the MLE $\hat{\lambda}$.

6. (L &K 6.14) What difficulty arises when you try to define a Q-Q plot for a discrete distribution?

7. (L &K 6.16) Suppose that the true distribution function $F(x)$ and the fitted distribution function $\hat{F}(x)$ are the same. For what distribution $F(x)$ will the Q-Q and P-P plots be essentially the same if the sample size is large?

8. (L &K 6.17) Suppose that the random variable M_j is the number of the n X_i's that would fall in the jth interval $[a_{j-1}, a_j)$ for a chi-square test if the fitted distribution were in fact the true one. What is the distribution of M_j, and what is its mean?

9. (L &K 6.21) Suppose that the data in the following table are independent observations on deviations from the desired diameter of ball bearings produced by a new high-speed machine. Use all appropriate techniques to hypothesize a distribution form, estimate its parameters (using MLEs), and evaluate goodness of fit.

Data on errors in the diameter of ball bearings

2.31	0.94	1.55	1.10	1.68	−0.16	0.48
1.49	1.20	1.48	0.85	3.21	1.71	4.01
2.10	0.26	1.97	1.09	2.72	1.18	0.28
0.30	1.40	0.59	1.99	2.14	1.59	1.50
0.48	2.12	1.15	2.54	0.70	1.63	1.47
1.71	1.41	0.95	1.55	1.28	0.44	−1.72
0.19	2.73	0.45	0.49	1.23	2.44	−1.62
0.00	1.33	−0.51	1.62	0.06	2.20	1.87
0.66	0.26	2.36	2.40	1.00	2.30	1.74
−1.27	3.11	1.03	0.59	1.37	1.30	0.78
1.01	0.99	0.24	2.18	2.24	0.22	1.01
−0.54	0.24	2.66	1.14	1.06	1.09	1.63
1.70	1.35	1.00	1.21	1.75	3.27	1.62
2.58	0.60	0.19	1.43	2.21	0.49	0.46
0.56	1.17	2.28	2.02	1.71	1.08	2.08
0.38	1.12	0.01	1.82	1.96	0.77	1.70
0.77	2.79	0.31	1.11	1.69	1.23	2.05
2.29	0.17	−0.12	2.69	1.78	2.26	0.02
1.55	0.44	0.89	1.51	−0.67	1.06	−0.05
0.27	0.78	0.60	1.06	2.29	1.13	1.85
1.62	1.50	0.21	2.04	1.26	1.98	1.50
0.94	0.17	1.90	1.64	1.12	0.89	0.49