AMS 553: Homework 5

1. (L&K 9.1) Argue heuristically that comparable output random variables from replications using different random numbers should be independent.

2. (L&K 9.3) In Example 9.9, suppose that the condition (b) is violated. In particular, suppose that it takes workers 20 minutes to put their tools away at the end of a shift and it takes the new workers 20 minutes to set up their tools at the beginning of the next shift. Does $N_1, N_2, \ldots$ have a steady-state distributions?

3. (L &K 9.9) Let p be a probability of interest for a terminating simulation. Define iid random variables $Y_1, Y_2, \ldots, Y_n$ such that $\hat{p} = \bar{Y}(n)$ and use these Y_j's in Eqs. (4.3), (4.4), and (4.12) to derive one possible confidence interval for p. Show that the variance estimate given by (4.4) can be written as $\hat{p}(1-\hat{p})/(n-1)$.

4. (L &K 9.11) For the $M/M/1$ queue with $\rho < 1$ of Example 9.23, suppose that the number of customers present when the first customer arrives has the following discrete distribution:

$$p(x) = (1-\rho)\rho^x \qquad \text{for } x = 0, 1, \ldots$$

   which is the steady state distribution of the number of customers in the system. Compute the distribution function of D_1 and its mean. In this case, it can be shown that D_i $i \geq 2$ has this same distribution.

5. (L &K 9.21) For the regenerative method, show that $\mu = E(Z)/E(N)$, where $\mu = lim_{i \to \infty} E[Y_i]$. (Hint: observe that

$$\frac{\sum_{j=1}^{n'} Z_j}{\sum_{j=1}^{n'} N_j} = \frac{\sum_{i=1}^{M(n')} Y_i}{M(n')},$$

   where n' is the number of regeneration cycles and $M(n')$ is the total number of observations in the n' cycles.) Also show that $\hat{\mu}(n') := \bar{Z}(n')/\bar{N}(n') \to \mu$ as $n' \to \infty$ w.p.1.

6. (L &K 9.22) For the queueing system considered in Example 9.30, are the indices of those customers who depart and leave exactly l customers behind ($l \geq 0$ and fixed) regeneration points for the process $D_1, D_2, \ldots$? If not, under what circumstances would they be?

7. (L &K 9.31) Let E_s be an event that occurs with probability $1 - \alpha_s$ for $s = 1, 2, \ldots, k$. Prove that

$$P\Big(\cap_{s=1}^{k} E_s \Big) \geq 1 - \sum_{s=1}^{k} \alpha_s$$

   .