AMS 151 (Fall, 2009) Joe Mitchell

Applied Calculus I
Practice Final Exam

IN ORDER TO RECEIVE FULL CREDIT, YOU MUST SHOW YOUR WORK AND GIVE YOUR REASONING. NO CREDIT WILL BE GIVEN FOR A NUMERICAL ANSWER WITHOUT SOME WRITTEN EXPLANATION!!

You do not need to evaluate expressions to a final answer: For example, you can leave an answer like "$x = 3 * 17 + \frac{4\ln(3\pi)}{\sqrt{3}} - \log_{10} 3$". Thus, **NO CALCULATORS ARE ALLOWED**.

CELL PHONES must be OFF (NOT on vibrate alert), and put out of sight If you have no watch and need updates of the time, please ask.

Last Name (PRINT CLEARLY): _____

First Name (PRINT CLEARLY): _____

ID Number: _____

PLEASE THINK ABOUT THE STATEMENT BELOW BEFORE YOU SIGN!!

Academic integrity is expected of all students at all times. Cheating will *not* be tolerated in this course. Students observing any act of academic dishonesty are requested to notify the instructor (discretely and anonymously at an appropriate time, in order to protect privacy).

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I cheat I will be subjected to the maximum possible penalty permitted under University guidelines, including dismissal from the University. I understand that the instructor will do everything in his power to make certain that the maximum applicable penalty is imposed.

Signature:

_____

---

1. Solve for x: $10 \cdot 3^{2x} = 3 \cdot (1.12)^x$

2. Determine which function has a larger value as $x \to \infty$:
   (a). $f(x) = 0.0005 \cdot x^3$ or $g(x) = 52,000 \cdot 2^x$

   (b). $f(x) = \log x^7$ or $g(x) = 6 \cdot \log^2 x^2$

3. Find the equation of the line that goes through the point (8,3) and is perpendicular to the plot of the equation $\frac{2x+1}{y-1} = 2$.

4. The quantity of moisture in a loaf of bread decreases with time as it sits on the table. Suppose that the moisture, $M(t)$, at time t minutes after being placed on the table decreases according to the function $M(t) = Qe^{-kt}$. If 10% of the moisture is gone at the end of 1 hour,
   (a). What percentage of the original moisture is present after 30 minutes?
   (b). How long will it take until the moisture is reduced to 60% of its original quantity?

5. Evaluate the following expressions:

(a). $\cos(\sin^{-1}(\frac{5}{11})) =$

(b). $\cos(\frac{32\pi}{3}) =$

6. On February 10, 1990, high tide in Boston was at midnight. The water level at high tide was 9.9 feet; later, at low tide, it was 0.1 feet. Assuming the next high tide is at exactly 12 noon and that the height of the water is given by a sine or cosine curve, find a formula for the water level (in feet) in Boston as a function of time t, measured in hours from midnight.

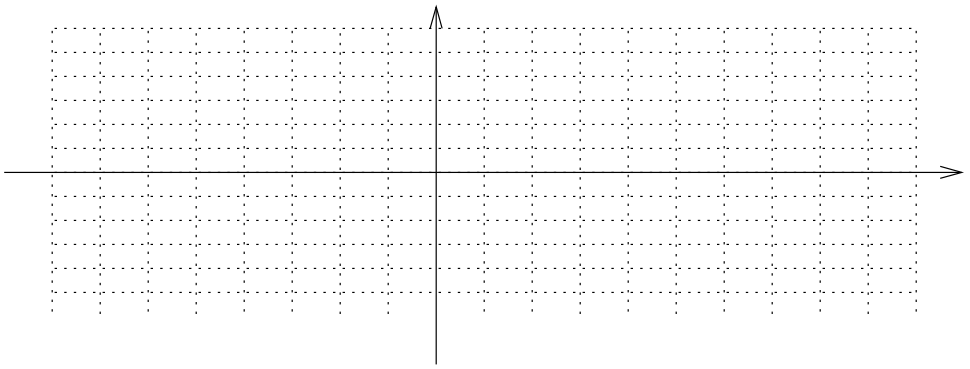
7. Find the asymptotes for the following function:

$$y = \frac{13 - 9x + 5x^2}{2x^2 - 1}$$

(a). Vertical:

(b). Horizontal:

(c). Sketch a plot of $y(x)$



8. The displacement (in feet) of a car moving in a straight line is given by $s(t) = 4t^2 + t + 1$, where t is measured in seconds.

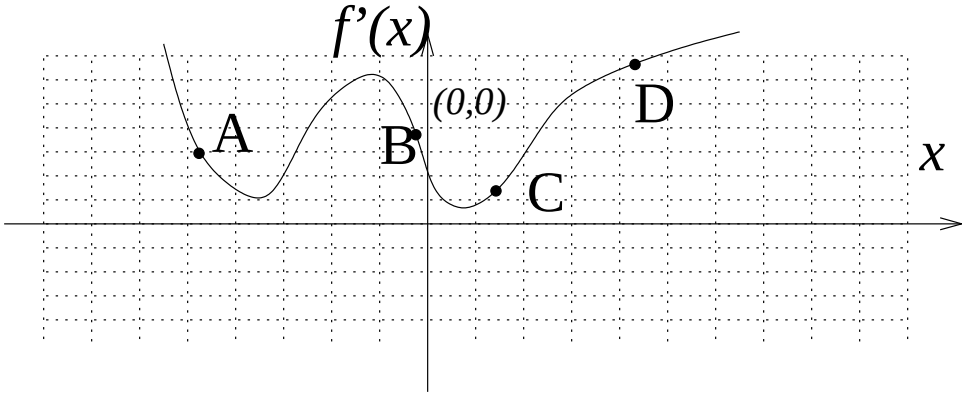
(a). Find the average velocity of the car over the time interval [9,13].

(b). Find the instantaneous velocity of the car when $t = 10$.

9. Find the equation of the tangent line to the curve $y - \sqrt{x} = 3$ $(x \geq 0)$ at the point (4,5).

10. If $h(b) = x^2b^3 + 3b^2 - \frac{x^2}{b^3}$, find $h'(1)$. Also find $h''(x)$.

11. The graph of $f'(x)$ is shown below. At which of the marked points is (a). $f(x)$ the least? (b). $f(x)$ the greatest? (c). $f'(x)$ the least? (d). $f'(x)$ the greatest? (e). $f''(x)$ the least? (f). $f''(x)$ the greatest?



12. At a time t after it is thrown up in the air, a ball is at a height of $f(t) = ct^2 + 8t + 3$ meters, where c is a constant. We are told that at time $t = 1$ the ball is accelerating *downwards* at 6 meters/sec^2.

(a). What is the velocity of the ball at time $t = 1$? Is the ball going up or going down at time $t = 1$?

(b). How high does the ball go?

13. (a). Let $f(x) = \frac{x^2 e^{-x}}{2x^3 + e^x} - \ln(\sqrt{x})$. Find $f'(x)$.
(b). Let $f(x) = \cos(3\tan(3x))$. Find $f'(x)$.

14. Suppose $xy - y^2 = e^{x^2}$. Compute $\frac{dy}{dx}$.

15. (a). Suppose that $h(x) = 2x + g(x^2)$ and that $g'(u) = 3u^2$. Find $h'(x)$.
(b). Let $f(x) = 3e^{2x+1}$. Find the 100th derivative, $f^{(100)}(x)$.

16. The average cost per item, C, in dollars, of manufacturing a quantity q of cell phones is given by $C = (a/q) + b$, where a, b, are positive constants. (a). Find the rate of change of C as q increases. What are its units? (b). If production increases at a rate of 100 cell phones per week, how fast is the average cost changing? Is the average cost increasing or decreasing?

17. (a). Compute $\lim_{t\to+\infty} 12te^{1/t} - 12t$.
(b). Compute $\lim_{t\to 0} \frac{1}{t} - \frac{1}{\sin t}$.

18. A rectangular beam is cut from a cylindrical log of radius 30 cm. The strength of a beam of width w and height h is proportional to wh^2. Find the width and height of the beam of maximum strength.

19. Consider the function $f(x) = 3x^5 - 5x^3$.
(a). Determine the critical points and classify each as a local max, local min, or neither.

(b). On which intervals is the function concave up? Concave down? Identify any inflection points.

(c). Find the global max/min on the interval $x \in [-10, 10]$.

20. Joe runs a marathon. He gets more and more tired, so his speed decreases over time. His speed at certain times is given below.

Time (min)	0	15	30	45	60	75	90
Speed (mph)	12	12	11	10	9	8	0

Give upper and lower bounds on the distance that Joe runs during the first hour.