Applied Calculus I
Practice Problems for Quiz # 2 – Solution Notes

1. Solve for x: $4 \cdot 2^{2x-1} - 3 \cdot 5^x = 0$.

$$4 \cdot 2^{2x-1} = 3 \cdot 5^x$$

$$\ln 4 + \ln 2^{2x-1} = \ln 3 + \ln 5^x$$

$$\ln 4 + (2x-1)\ln 2 = \ln 3 + x \ln 5$$

$$(2\ln 2 - \ln 5)x = \ln 3 - \ln 4 - \ln 2$$

$$x = \frac{\ln 3 - \ln 4 - \ln 2}{2\ln 2 - \ln 5} = \frac{\ln(3/8)}{\ln(4/5)}$$

2. Determine which function has a larger value as $x \to \infty$:
(a). $f(x) = 5 \cdot x^2 - 57x$ or $g(x) = 0.002 \cdot (1.001)^x$

$g(x)$ grows faster, since is is exponential (proportional to 1.001^x), while $f(x)$ is polynomial (grows like x^2).

(b). $f(x) = \log^4 x$ or $g(x) = 3 \cdot \log^2 x^3$ $f(x)$ grows faster, since it grows like $\log^4 x$, while $g(x)$ grows like $\log^2 x$ (a lower power of logarithm).

3. Find the inverse function of $y = g(t) = 3e^{t+4} - 2$.

$$\frac{2+y}{3} = e^{t+4}$$

$$\ln(\frac{2+y}{3}) = t + 4$$

$$t = \ln(\frac{2+y}{3}) - 4$$

Thus, $g^{-1}(y) = \ln(\frac{2+y}{3}) - 4$.

4. Let $y = h(x) = \frac{2}{3-e^{-x^2}}$.
(a). Is h increasing or decreasing?

x^2 is increasing, so $-x^2$ is decreasing, so e^{-x^2} is decreasing, so $-e^{-x^2}$ is increasing, so $\frac{2}{3-e^{-x^2}}$ is decreasing.

(b). Find a formula for $h^{-1}(u)$.

Actually, since $h(x)$ is even ($h(-x) = h(x)$), there is no inverse. However, we can find two "branches" of the inverse function, as follows:

$$y(3 - e^{-x^2}) = 2$$

$$3 - e^{-x^2} = \frac{2}{y}$$

$$3 - \frac{2}{y} = e^{-x^2}$$

$$\ln(3 - \frac{2}{y}) = -x^2$$

$$x^2 = -\ln(3 - \frac{2}{y})$$

$$x = \pm\sqrt{-\ln(3 - \frac{2}{y})}$$

Note that we must have $0 < 3 - \frac{2}{y} \leq 1$, so that the expression within the square root is nonnegative, since $\ln w \leq 0$ for $0 < w \leq 1$.

We can define the inverse to be either of the two branches of the square root function; e.g., $h^{-1}(y) = \sqrt{-\ln(3 - \frac{2}{y})}$ (so that $h^{-1}(u) = \sqrt{-\ln(3 - \frac{2}{u})}$).

5. As a new-born infant, Joe was exposed to radioactive strontium-90 in 1968. The half-life of strontium-90 is 29 years. From age 10 to age 20, what was the percentage decrease in the quantity of strontium-90 in Joe's bones? (For example, if Joe had 20 mg in his bones at age 10 and had 15 mg in his bones at age 20, there was a 25% decrease in the quantity, from 20 mg to 15 mg.)

Let $f(t)$ be the amount of strontium-90 in Joe's bones when he is t years old.
We model: $f(t) = Qe^{-kt}$.
We know: $f(29) = \frac{1}{2}Q$, thus, $Qe^{-29k} = \frac{1}{2}$, so $-29k = \ln\frac{1}{2}$, so $k = -\frac{1}{29}\ln\frac{1}{2}$.
We want the percent decrease in going from $f(10)$ down to $f(20)$, i.e.,

$$\frac{f(10) - f(20)}{f(10)} \cdot 100 = \frac{Qe^{-10k} - Qe^{-20k}}{Qe^{-10k}} \cdot 100$$

$$= (1 - e^{-20k+10k}) \cdot 100 = (1 - e^{-10k}) \cdot 100 = (1 - e^{-10 \cdot \frac{-1}{29}\ln\frac{1}{2}}) \cdot 100 = (1 - (\frac{1}{2})^{10/29}) \cdot 100$$

6. Find the equation of a line of slope 1 that is at distance 1 from the origin? (The distance from the origin to a line L is the distance from the point (0,0) to the point, p, on L that is closest to (0,0) among all points on L.)
How many such lines are there?

Two lines satisfy this: $y = x - \sqrt{2}$, $y = x + \sqrt{2}$. (Draw a picture!)

7. Let $f(x) = 2^x$ and $g(x) = x^3 - 7$.
(a). Determine $g(f(y))$.

$$g(f(y)) = g(2^y) = (2^y)^3 - 7 = 2^{3y} - 7$$

(b). Which function has a larger value as $x \to \infty$: $g(f(x))$ or $f(g(x))$?

$g(f(x)) = 2^{3x} - 7$, as we saw in part (a).
Similarly, $f(g(x)) = f(x^3 - 7) = 2^{x^3-7}$.
Thus, $f(g(x))$ grows faster (it grows like 2^{x^3}, while $g(f(x))$ grows like 2^{3x}: x^3 grows much faster than $3x$).

8. Plot $f(x) = -3\cos(x - \pi)$. Be sure to mark important points on the axes!

Use your graphing calculator (or Wolfram alpha, or similar) to check that you can plot it!